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ECONOMIC INVESTIGATION OF UTILITY MAXIMIZATION THROUGH MATHEMATICAL ANALYSIS

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Abstract: *In this paper economic predictions among commodity prices and Lagrange multipliers are deliberated, where utility maximization policy is examined with sufficient mathematical investigation. In mathematical economics, utility reflects the propensity of an activity, which of course increases or decreases complete happiness of the individual/society. The system of Lagrange multiplier is a strongly valuable method in advanced calculus that is applied in this paper with a pair of constraints. This paper also tries to show the economic results of optimization precisely but elaborately.*

Keywords: Commodity price, economic predictions, utility maximization

JEL Codes: C61, C67, D21, D24, H32, I31

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1. Introduction

"Mathematical modeling" is thought as a very important research area in modern economics [Samuelson, 1947]. It is an extremely theoretical and analytical research that covers many areas of sociology, such as economics, social science, psychology, behavioral science, political science, etc. [Zheng & Liu, 2022]. Mathematical modeling of utility maximization is necessary to find accurate outcome in sensitivity analysis [Gauthier, 1975]. In economics, Lagrange multiplier analysis is a popular mathematical policy [Islam et al., 2009a; Mohajan, 2021b].

The economic condition is such that after the use of commodities, can able to satisfy wishes of humanity is considered as utility [Bentham, 1780]. Utility is an important and fundamental concept in modern economics [Fishburn, 1970]. The idea of utility is settled in economics during the 18th century by philosophers Jeremy Bentham (1748-1832) and John Stuart Mill (1806-1873) (Bentham, 1780]. They have realized that utility can raise or reduce the total happiness [Read, 2004]. Activities of utility indicate that citizens of the society try to find maximum satisfaction from their buying materials. Producers in the society always depend on the consumer's satisfaction of their products [Kirsh, 2017]. Throughout the study mathematical calculations are given in some details.

2. Literature Review

In any research, the literature review indicates works of former researchers considers their prevailing facts [Polit & Hungler, 2013]. Charles W. Cobb (1875-1949) and Paul H. Douglas (1892-1976) obtains a function related to industrial production [Cobb & Douglas, 1928]. John V. Baxley and John C. Moorhouse analyze the optimization policy [Baxley & Moorhouse, 1984]. Well-known Professor Jamal Nazrul Islam (1939-2013) and his collaborators work on utility maximization policy in mathematical economics [Islam et al., 2010, 2011]. Pahlaj Moolio and his associates show the optimization with detail mathematical analysis [Moolio et al., 2009]. Lia Roy and her partners discuss the cost minimization with detail mathematical presentation [Roy et al., 2021].

Devajit Mohajan and Haradhan Kumar Mohajan work on profit maximization area, and they have revealed mathematical operations very elaborately [Mohajan & Mohajan, 2022a-d]. Later, Jannatul Ferdous and Haradhan Kumar Mohajan take endeavors for solving optimization relations [Ferdous & Mohajan, 2022]. Recently, Sabo Nelson Pandi and his colleagues have displayed lifetime utility maximization structure [Pandi et al., 2022].

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3. Research Methodology

Research is a systematic inquiry to achieve new knowledge with the existing evidences. It tries to remove existing mistakes and misconceptions, and adds new knowledge with the present stock of knowledge [Pandey & Pandey, 2015]. Methodology is the scientific actions, techniques and instruments, and also a guideline to perform an acceptable research. It supports the new researchers to accomplish better research [Kothari, 2008]. Research methodology is a way for thoroughly solving the research activities (Legesse, 2014). A researcher faces numerous problems during data collection, statistical designs, and finally to prepare a seminal research work [Mohajan, 2011, 2012b,c, 2013a,b,c, 2014, 2017b,d,e, 2018a,b, 2020, 2021f].

We have started the paper with four commodity variables: M_1, M_2, M_3, and M_4 to predict in the economic optimization on the basis of the results of sensitivity analysis. We have discussed sensitivity analysis using $\frac{\partial \lambda_1}{\partial p_1}$, $\frac{\partial \lambda_2}{\partial p_4}$, etc. [Mohajan & Datta, 2012]. We have prepared this paper by consulting the books of famous authors, journals, handbooks, theses, and by taking the help from the internet, websites, etc. [Mohajan & Mohajan, 2023a-o].

4. Objective of the Study

Principal objective of the article is to discuss economic predictions among Lagrange multipliers and commodity prices during utility maximization observation. Some more subsidiary objectives are;

- to demonstrate the mathematical thoughts accurately,
- to give the economic predictions properly, and
- to encourage the young researchers in mathematical economic modeling research areas.

5. A Mathematical Model for Utility

To work on economic predictions we consider that there are only four commodities in the society, such as M_1, M_2, M_3, and M_4. Let a person buys m_1, m_2, m_3, and m_4 quantities from these four goods M_1, M_2, M_3, and M_4,

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respectively. The utility function is [Islam et al., 2010; Mohajan & Mohajan, 2022b],

$$U(m_1, m_2, m_3, m_4) = m_1 m_2 m_3 m_4 . \quad (1)$$

The budget constraint will be,

$$B(m_1, m_2, m_3, m_4) = p_1 m_1 + p_2 m_2 + p_3 m_3 + p_4 m_4 \quad (2)$$

where p_1, p_2, p_3, and p_4 are the prices of per unit of goods m_1, m_2, m_3, and m_4, respectively. The coupon constraint is,

$$\Pi(m_1, m_2, m_3, m_4) = \pi_1 m_1 + \pi_2 m_2 + \pi_3 m_3 + \pi_4 m_4 , \quad (3)$$

where π_1, π_2, π_3, and π_4 are the coupons necessary to purchase a unit of commodity of m_1, m_2, m_3, and m_4, respectively.

Now we use relations (1) to (3) to present Lagrangian function $\Lambda(m_1, m_2, m_3, m_4, \lambda_1, \lambda_2)$ for Lagrange multipliers λ_1 and λ_2 as [Baxley & Moorhouse, 1984; Ferdous & Mohajan, 2022],

$$\Lambda(m_1, m_2, m_3, m_4, \lambda_1, \lambda_2) = m_1 m_2 m_3 m_4 + \lambda_1 (B - p_1 m_1 - p_2 m_2 - p_3 m_3 - p_4 m_4) + \lambda_2 (\Pi - \pi_1 m_1 - \pi_2 m_2 - \pi_3 m_3 - \pi_4 m_4) . \quad (4)$$

Now we apply techniques of calculus in (4) to find [Islam et al. 2010; Mohajan & Mohajan, 2022c];

$$B_1 = p_1 , \; B_2 = p_2 , B_3 = p_3 , \; B_4 = p_4 ;$$
$$\Pi_1 = \pi_1 , \Pi_2 = \pi_2 , \quad \Pi_3 = \pi_3 , \quad \Pi_4 = \pi_4 ; \quad (5)$$
$$\Lambda_{11} = 0 , \; \Lambda_{12} = \Lambda_{21} = m_3 m_4 , \; \Lambda_{13} = \Lambda_{31} = m_2 m_4 ,$$
$$\Lambda_{14} = \Lambda_{41} = m_2 m_3 , \quad \Lambda_{22} = 0 , \Lambda_{23} = \Lambda_{32} = m_1 m_4 ,$$
$$\Lambda_{24} = \Lambda_{42} = m_1 m_3 , \quad \Lambda_{33} = 0 , \; \Lambda_{34} = \Lambda_{43} = m_1 m_2 , \quad \Lambda_{44} = 0 , \quad (6)$$

where $\frac{\partial B}{\partial m_1} = B_1$, $\frac{\partial B}{\partial m_2} = B_2$, $\frac{\partial \Pi}{\partial m_1} = \Pi_1$, $\frac{\partial \Pi}{\partial m_2} = \Pi_2$, $\frac{\partial \Lambda}{\partial m_1} = \Lambda_1$, $\frac{\partial \Lambda}{\partial m_2 \partial m_3} = \Lambda_{32}$, $\frac{\partial^2 \Lambda}{\partial m_2^2} = \Lambda_{22}$, etc. are partial differentiations of multivariate functions. We have the bordered Hessian [Islam et al., 2011a; Mohajan & Mohajan, 2022c],

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$$|H| = \begin{vmatrix} 0 & 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & 0 & -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ -B_1 & -\Pi_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -B_2 & -\Pi_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -B_3 & -\Pi_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -B_4 & -\Pi_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix}. \quad (7)$$

Simplify (7) we obtain [Mohajan & Mohajan, 2023a];

$$|H| = -2p_1p_2\pi_1\pi_2 < 0. \quad (8)$$

We obtain Lagrange multiplier $\lambda_1 > 0$ as [Mohajan & Mohajan, 2023b],

$$\lambda_1 = m_3m_4\frac{m_1\pi_1 - m_2\pi_2}{\pi_1p_2 - \pi_2p_1} \quad (9)$$

where $\pi_1p_2 \neq \pi_2p_1$.

We find $\lambda_2 > 0$ as [Mohajan & Mohajan, 2023e],

$$\lambda_2 = m_3m_4\frac{m_1p_1 - m_2p_2}{\pi_2p_1 - \pi_1p_2} \quad (10)$$

where $\pi_2p_1 - \pi_1p_2 \neq 0$.

For m_1, m_2, m_3, m_4, λ_1, and λ_2 in terms of p_1, p_2, p_3, p_4, π_1, π_2, π_3, π_4, B, and Π, we can find $\frac{\partial\lambda_1}{\partial p_1}$, $\frac{\partial\lambda_2}{\partial p_1}$, …, $\frac{\partial\lambda_1}{\partial\pi_1}$, $\frac{\partial\lambda_2}{\partial\pi_1}$, …, $\frac{\partial m_1}{\partial p_1}$, …, $\frac{\partial m_1}{\partial\pi_1}$, …, $\frac{\partial\lambda_1}{\partial B}$, …, $\frac{\partial\lambda_1}{\partial\Pi}$, etc., [Islam et al., 2011; Mohajan, 2021c]. Also we have [Mohajan, 2021b; Mohajan & Mohajan, 2023c];

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$$J = H = \begin{vmatrix} 0 & 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & 0 & -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ -B_1 & -\Pi_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -B_2 & -\Pi_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -B_3 & -\Pi_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -B_4 & -\Pi_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix} \tag{11}$$

which must be non-singular. Now we apply "The Implicit Function Theorem" as [Moolio et al., 2009; Islam et al., 2010];

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \mathbf{G}\left(p_1, p_2, p_3, p_4, \pi_1, \pi_2, \pi_3, \pi_4, B, \Pi\right). \tag{12}$$

Now the 6×10 Jacobian matrix for $\mathbf{G}$; regarded as $J_G = \dfrac{\partial(\lambda_1, \lambda_2, m_1, m_2, m_3, m_4)}{\partial(p_1, p_2, p_3, p_4, \pi_1, \pi_2, \pi_3, \pi_4, B, \Pi)}$, and is presented as [Mohajan, 2021a; Mohajan & Mohajan, 2023d],

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$$J_G = \begin{bmatrix} \frac{\partial \lambda_1}{\partial p_1} & \frac{\partial \lambda_1}{\partial p_2} & \frac{\partial \lambda_1}{\partial p_3} & \frac{\partial \lambda_1}{\partial p_4} & \frac{\partial \lambda_1}{\partial \pi_1} & \frac{\partial \lambda_1}{\partial \pi_2} & \frac{\partial \lambda_1}{\partial \pi_3} & \frac{\partial \lambda_1}{\partial \pi_4} & \frac{\partial \lambda_1}{\partial B} & \frac{\partial \lambda_1}{\partial \Pi} \\ \frac{\partial \lambda_2}{\partial p_1} & \frac{\partial \lambda_2}{\partial p_2} & \frac{\partial \lambda_2}{\partial p_3} & \frac{\partial \lambda_2}{\partial p_4} & \frac{\partial \lambda_2}{\partial \pi_1} & \frac{\partial \lambda_2}{\partial \pi_2} & \frac{\partial \lambda_2}{\partial \pi_3} & \frac{\partial \lambda_2}{\partial \pi_4} & \frac{\partial \lambda_2}{\partial B} & \frac{\partial \lambda_2}{\partial \Pi} \\ \frac{\partial m_1}{\partial p_1} & \frac{\partial m_1}{\partial p_2} & \frac{\partial m_1}{\partial p_3} & \frac{\partial m_1}{\partial p_4} & \frac{\partial m_1}{\partial \pi_1} & \frac{\partial m_1}{\partial \pi_2} & \frac{\partial m_1}{\partial \pi_3} & \frac{\partial m_1}{\partial \pi_4} & \frac{\partial m_1}{\partial B} & \frac{\partial m_1}{\partial \Pi} \\ \frac{\partial m_2}{\partial p_1} & \frac{\partial m_2}{\partial p_2} & \frac{\partial m_2}{\partial p_3} & \frac{\partial m_2}{\partial p_4} & \frac{\partial m_2}{\partial \pi_1} & \frac{\partial m_2}{\partial \pi_2} & \frac{\partial m_2}{\partial \pi_3} & \frac{\partial m_2}{\partial \pi_4} & \frac{\partial m_2}{\partial B} & \frac{\partial m_2}{\partial \Pi} \\ \frac{\partial m_3}{\partial p_1} & \frac{\partial m_3}{\partial p_2} & \frac{\partial m_3}{\partial p_3} & \frac{\partial m_3}{\partial p_4} & \frac{\partial m_3}{\partial \pi_1} & \frac{\partial m_3}{\partial \pi_2} & \frac{\partial m_3}{\partial \pi_3} & \frac{\partial m_3}{\partial \pi_4} & \frac{\partial m_3}{\partial B} & \frac{\partial m_3}{\partial \Pi} \\ \frac{\partial m_4}{\partial p_1} & \frac{\partial m_4}{\partial p_2} & \frac{\partial m_4}{\partial p_3} & \frac{\partial m_4}{\partial p_4} & \frac{\partial m_4}{\partial \pi_1} & \frac{\partial m_4}{\partial \pi_2} & \frac{\partial m_4}{\partial \pi_3} & \frac{\partial m_4}{\partial \pi_4} & \frac{\partial m_4}{\partial B} & \frac{\partial m_4}{\partial \Pi} \end{bmatrix}. \quad (13)$$

$$= -J^{-1} \begin{bmatrix} -m_1 & -m_2 & -m_3 & -m_4 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -m_1 & -m_2 & -m_3 & -m_4 & 0 & 1 \\ -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 \end{bmatrix}. \quad (14)$$

The inverse of Jacobian matrix is, $J^{-1} = \frac{1}{|J|} C^T$, where $C = \left(C_{ij}\right)$, and (14) gives [Mohajan, 2017a; Islam et al., 2009b, 2011],

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$$J_G = -\frac{1}{|J|} C^T \begin{bmatrix} -m_1 & -m_2 & -m_3 & -m_4 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -m_1 & -m_2 & -m_3 & -m_4 & 0 & 1 \\ -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 \end{bmatrix} . \quad (15)$$

Now 6×6 transpose matrix C^T can be represented by,

$$C^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} & C_{61} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} & C_{62} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} & C_{63} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} & C_{64} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{65} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} . \quad (16)$$

Using (16) we develop (13) as [Mohajan & Mohajan, 2022b];

$$J_G = -\frac{1}{|J|} \begin{bmatrix} -m_1C_{11}-\lambda_1C_{31} & -m_2C_{11}-\lambda_1C_{41} & -m_3C_{11}-\lambda_1C_{51} & -m_4C_{11}-\lambda_1C_{61} & -m_1C_{21}-\lambda_2C_{31} \\ -m_1C_{12}-\lambda_1C_{32} & -m_2C_{12}-\lambda_1C_{42} & -m_3C_{12}-\lambda_1C_{52} & -m_4C_{12}-\lambda_1C_{62} & -m_1C_{22}-\lambda_2C_{32} \\ -m_1C_{13}-\lambda_1C_{33} & -m_2C_{13}-\lambda_1C_{43} & -m_3C_{13}-\lambda_1C_{53} & -m_4C_{13}-\lambda_1C_{63} & -m_1C_{23}-\lambda_2C_{33} \\ -m_1C_{14}-\lambda_1C_{34} & -m_2C_{14}-\lambda_1C_{44} & -m_3C_{14}-\lambda_1C_{54} & -m_4C_{14}-\lambda_1C_{64} & -m_1C_{24}-\lambda_2C_{34} \\ -m_1C_{15}-\lambda_1C_{35} & -m_2C_{15}-\lambda_1C_{45} & -m_3C_{15}-\lambda_1C_{55} & -m_4C_{15}-\lambda_1C_{65} & -m_1C_{25}-\lambda_2C_{35} \\ -m_1C_{16}-\lambda_1C_{36} & -m_2C_{16}-\lambda_1C_{46} & -m_3C_{16}-\lambda_1C_{56} & -m_4C_{16}-\lambda_1C_{66} & -m_1C_{26}-\lambda_2C_{36} \end{bmatrix}$$

$$\begin{bmatrix} -m_2C_{21}-\lambda_2C_{41} & -m_3C_{21}-\lambda_2C_{51} & -m_4C_{21}-\lambda_2C_{61} & C_{11} & C_{21} \\ -m_2C_{22}-\lambda_2C_{42} & -m_3C_{22}-\lambda_2C_{52} & -m_4C_{22}-\lambda_2C_{62} & C_{12} & C_{22} \\ -m_2C_{23}-\lambda_2C_{43} & -m_3C_{23}-\lambda_2C_{53} & -m_4C_{23}-\lambda_2C_{63} & C_{13} & C_{23} \\ -m_2C_{24}-\lambda_2C_{44} & -m_3C_{24}-\lambda_2C_{54} & -m_4C_{24}-\lambda_2C_{64} & C_{14} & C_{24} \\ -m_2C_{25}-\lambda_2C_{45} & -m_3C_{25}-\lambda_2C_{55} & -m_4C_{25}-\lambda_2C_{65} & C_{15} & C_{25} \\ -m_2C_{26}-\lambda_2C_{46} & -m_3C_{26}-\lambda_2C_{56} & -m_4C_{26}-\lambda_2C_{66} & C_{16} & C_{26} \end{bmatrix} . \quad (17)$$

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6. Sensitivity Analysis

Now we try with λ_1 when unit price p_1 of commodity m_1 upsurges. Considering T_{11} we can write [Islam et al., 2011; Mohajan & Mohajan, 2023c],

$$\frac{\partial \lambda_1}{\partial p_1} = \frac{m_1}{|J|}[C_{11}] + \frac{\lambda_1}{|J|}[C_{31}]$$

$$= \frac{m_1}{|J|}\text{Cofactor of } C_{11} + \frac{\lambda_1}{|J|}\text{Cofactor of } C_{31}$$

$$= \frac{m_1}{|J|}\begin{vmatrix} 0 & -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ -\Pi_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -\Pi_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix}$$

$$+ \frac{\lambda_1}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ -\Pi_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix}$$

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$$= \frac{m_1}{|J|}\left\{ \Pi_1 \begin{vmatrix} -\Pi_1 & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -\Pi_2 & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix} - \Pi_2 \begin{vmatrix} -\Pi_1 & \Lambda_{11} & \Lambda_{13} & \Lambda_{14} \\ -\Pi_2 & \Lambda_{21} & \Lambda_{23} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{31} & \Lambda_{33} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{41} & \Lambda_{43} & \Lambda_{44} \end{vmatrix} \right.$$

$$\left. + \Pi_3 \begin{vmatrix} -\Pi_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{14} \\ -\Pi_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{44} \end{vmatrix} - \Pi_4 \begin{vmatrix} -\Pi_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ -\Pi_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ -\Pi_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \\ -\Pi_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} \end{vmatrix} \right\}$$

$$+ \frac{\lambda_1}{|J|}\left\{ -\Pi_2 \begin{vmatrix} -B_1 & -B_2 & -B_3 & -B_4 \\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix} + \Pi_3 \begin{vmatrix} -B_1 & -B_2 & -B_3 & -B_4 \\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix} \right.$$

$$\left. - \Pi_4 \begin{vmatrix} -B_1 & -B_2 & -B_3 & -B_4 \\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \end{vmatrix} \right\}$$

$$= \frac{m_1}{|J|}\left[\Pi_1 \left\{ -\Pi_1 \begin{vmatrix} \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix} - \Lambda_{12} \begin{vmatrix} -\Pi_2 & \Lambda_{23} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{33} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{43} & \Lambda_{44} \end{vmatrix} + \Lambda_{13} \begin{vmatrix} -\Pi_2 & \Lambda_{22} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{32} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{42} & \Lambda_{44} \end{vmatrix} \right.\right.$$

$$\left.\left. - \Lambda_{14} \begin{vmatrix} -\Pi_2 & \Lambda_{22} & \Lambda_{23} \\ -\Pi_3 & \Lambda_{32} & \Lambda_{33} \\ -\Pi_4 & \Lambda_{42} & \Lambda_{43} \end{vmatrix} \right\} - \Pi_2 \left\{ -\Pi_1 \begin{vmatrix} \Lambda_{21} & \Lambda_{23} & \Lambda_{24} \\ \Lambda_{31} & \Lambda_{33} & \Lambda_{34} \\ \Lambda_{41} & \Lambda_{43} & \Lambda_{44} \end{vmatrix} + \Lambda_{13} \begin{vmatrix} -\Pi_2 & \Lambda_{21} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{31} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{41} & \Lambda_{44} \end{vmatrix} \right.$$

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$$-\Lambda_{14}\begin{vmatrix}-\Pi_2 & \Lambda_{21} & \Lambda_{23}\\ -\Pi_3 & \Lambda_{31} & \Lambda_{33}\\ -\Pi_4 & \Lambda_{41} & \Lambda_{43}\end{vmatrix}\Bigg\} \quad +\Pi_3\Bigg\{-\Pi_1\begin{vmatrix}\Lambda_{21} & \Lambda_{22} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{44}\end{vmatrix} \quad +\Lambda_{12}\begin{vmatrix}-\Pi_2 & \Lambda_{21} & \Lambda_{24}\\ -\Pi_3 & \Lambda_{31} & \Lambda_{34}\\ -\Pi_4 & \Lambda_{41} & \Lambda_{44}\end{vmatrix}$$

$$-\Lambda_{14}\begin{vmatrix}-\Pi_2 & \Lambda_{21} & \Lambda_{22}\\ -\Pi_3 & \Lambda_{31} & \Lambda_{32}\\ -\Pi_4 & \Lambda_{41} & \Lambda_{42}\end{vmatrix}\Bigg\} \quad -\Pi_4\Bigg\{-\Pi_1\begin{vmatrix}\Lambda_{21} & \Lambda_{22} & \Lambda_{23}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43}\end{vmatrix} \quad +\Lambda_{12}\begin{vmatrix}-\Pi_2 & \Lambda_{21} & \Lambda_{23}\\ -\Pi_3 & \Lambda_{31} & \Lambda_{33}\\ -\Pi_4 & \Lambda_{41} & \Lambda_{43}\end{vmatrix}$$

$$-\Lambda_{13}\begin{vmatrix}-\Pi_2 & \Lambda_{21} & \Lambda_{22}\\ -\Pi_3 & \Lambda_{31} & \Lambda_{32}\\ -\Pi_4 & \Lambda_{41} & \Lambda_{42}\end{vmatrix}\Bigg\}\Bigg] + \frac{\lambda_1}{|J|}\Bigg[-\Pi_2\Bigg\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{32} & \Lambda_{33} & \Lambda_{34}\\ \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{31} & \Lambda_{33} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{43} & \Lambda_{44}\end{vmatrix}$$

$$-B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{44}\end{vmatrix}$$

$$+B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43}\end{vmatrix}\Bigg\} \quad +\Pi_3\Bigg\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{21} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{41} & \Lambda_{43} & \Lambda_{44}\end{vmatrix}$$

$$-B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{24}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{44}\end{vmatrix} + B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43}\end{vmatrix}\Bigg\} \quad -\Pi_4\Bigg\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{32} & \Lambda_{33} & \Lambda_{34}\end{vmatrix}$$

$$+B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{21} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{33} & \Lambda_{34}\end{vmatrix} - B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{34}\end{vmatrix}$$

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$$+B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33}\end{vmatrix}\Bigg\}\Bigg]$$

$$=\frac{m_1}{|J|}\{-\Pi_1^2\Lambda_{23}\Lambda_{24}\Lambda_{34} \quad -\Pi_1^2\Lambda_{23}\Lambda_{24}\Lambda_{34} \quad -\Pi_1\Pi_2\Lambda_{12}\Lambda_{34}^2 \quad -\Pi_1\Pi_4\Lambda_{12}\Lambda_{23}\Lambda_{34}$$
$$+\Pi_1\Pi_3\Lambda_{12}\Lambda_{24}\Lambda_{34} \quad +\Pi_1\Pi_4\Lambda_{12}\Lambda_{23}\Lambda_{34} \quad +\Pi_1\Pi_2\Lambda_{13}\Lambda_{34}^2 \quad -\Pi_1\Pi_3\Lambda_{13}\Lambda_{24}^2$$
$$+\Pi_1\Pi_4\Lambda_{13}\Lambda_{23}\Lambda_{24} \quad +\Pi_1\Pi_2\Lambda_{14}\Lambda_{23}\Lambda_{34} \quad +\Pi_1\Pi_3\Lambda_{14}\Lambda_{23}\Lambda_{24} \quad -\Pi_1\Pi_4\Lambda_{14}\Lambda_{23}^2$$
$$-\Pi_1\Pi_2\Lambda_{12}\Lambda_{34}^2 \quad +\Pi_1\Pi_2\Lambda_{14}\Lambda_{23}\Lambda_{34} \quad +\Pi_1\Pi_2\Lambda_{13}\Lambda_{24}\Lambda_{34} \quad -\Pi_2^2\Lambda_{13}\Lambda_{14}\Lambda_{34}$$
$$+\Pi_2\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{34} \quad +\Pi_2\Pi_3\Lambda_{13}\Lambda_{14}\Lambda_{24} \quad -\Pi_2\Pi_4\Lambda_{13}^2\Lambda_{24} \quad -\Pi_2^2\Lambda_{13}\Lambda_{14}\Lambda_{34}$$
$$+\Pi_2\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{34} \quad -\Pi_2\Pi_3\Lambda_{14}^2\Lambda_{23} \quad +\Pi_2\Pi_4\Lambda_{13}\Lambda_{14}\Lambda_{23} \quad +\Pi_2\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{34}$$
$$-\Pi_3\Pi_4\Lambda_{12}^2\Lambda_{34} \quad -\Pi_3^2\Lambda_{12}\Lambda_{13}\Lambda_{24} \quad +\Pi_3\Pi_4\Lambda_{12}\Lambda_{14}\Lambda_{24} \quad +\Pi_2\Pi_3\Lambda_{13}\Lambda_{14}\Lambda_{24}$$
$$-\Pi_2\Pi_3\Lambda_{14}^2\Lambda_{23} \quad -\Pi_3^2\Lambda_{12}\Lambda_{14}^2 \quad +\Pi_3\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{14} \quad +\Pi_1\Pi_4\Lambda_{12}\Lambda_{23}\Lambda_{34}$$
$$+\Pi_1\Pi_4\Lambda_{13}\Lambda_{23}\Lambda_{24} \quad -\Pi_1\Pi_4\Lambda_{14}\Lambda_{23}^2 \quad +\Pi_2\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{34} \quad -\Pi_3\Pi_4\Lambda_{12}^2\Lambda_{34}$$
$$+\Pi_3\Pi_4\Lambda_{12}\Lambda_{14}\Lambda_{23} \quad -\Pi_4^2\Lambda_{12}\Lambda_{13}\Lambda_{23} \quad -\Pi_2\Pi_4\Lambda_{13}^2\Lambda_{24} \quad +\Pi_2\Pi_4\Lambda_{13}\Lambda_{14}\Lambda_{23}$$
$$+\Pi_3\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{24} \quad -\Pi_4^2\Lambda_{12}\Lambda_{13}\Lambda_{23}\} \quad +\frac{\lambda_1}{|J|}\{+B_1\Pi_2^2\Lambda_{34}^2 \quad -B_1\Pi_2\Pi_3\Lambda_{24}\Lambda_{34}$$
$$-B_1\Pi_2\Pi_4\Lambda_{23}\Lambda_{34} \quad -B_2\Pi_1\Pi_2\Lambda_{34}^2 \quad +B_2\Pi_2\Pi_3\Lambda_{14}\Lambda_{34} \quad +B_2\Pi_2\Pi_4\Lambda_{13}\Lambda_{34}$$
$$+B_3\Pi_1\Pi_2\Lambda_{24}\Lambda_{34} \quad -B_3\Pi_2^2\Lambda_{14}\Lambda_{34} \quad -B_3\Pi_2\Pi_4\Lambda_{13}\Lambda_{24} \quad +B_3\Pi_2\Pi_4\Lambda_{14}\Lambda_{23}$$
$$+B_4\Pi_1\Pi_2\Lambda_{23}\Lambda_{34} \quad -B_4\Pi_2^2\Lambda_{13}\Lambda_{34} \quad +B_4\Pi_2\Pi_3\Lambda_{13}\Lambda_{24} \quad -B_4\Pi_2\Pi_3\Lambda_{14}\Lambda_{23}$$
$$-B_1\Pi_2\Pi_3\Lambda_{24}\Lambda_{34} \quad +B_1\Pi_3^2\Lambda_{24}^2 \quad -B_1\Pi_3\Pi_4\Lambda_{23}\Lambda_{24} \quad -B_2\Pi_1\Pi_3\Lambda_{24}\Lambda_{34}$$
$$-B_2\Pi_3^2\Lambda_{14}\Lambda_{24} \quad -B_2\Pi_3\Pi_4\Lambda_{12}\Lambda_{34} \quad +B_2\Pi_3\Pi_4\Lambda_{14}\Lambda_{23} \quad -B_3\Pi_1\Pi_3\Lambda_{24}^2$$
$$+B_3\Pi_2\Pi_3\Lambda_{14}\Lambda_{24} \quad +B_3\Pi_3\Pi_4\Lambda_{12}\Lambda_{24} \quad +B_4\Pi_1\Pi_3\Lambda_{23}\Lambda_{24} \quad +B_4\Pi_2\Pi_3\Lambda_{12}\Lambda_{34}$$
$$-B_4\Pi_2\Pi_3\Lambda_{14}\Lambda_{23} \quad -B_4\Pi_3^2\Lambda_{12}\Lambda_{24} \quad -B_1\Pi_2\Pi_4\Lambda_{23}\Lambda_{34} \quad -B_1\Pi_3\Pi_4\Lambda_{23}\Lambda_{24}$$
$$+B_1\Pi_4^2\Lambda_{23}^2 \quad +B_2\Pi_1\Pi_4\Lambda_{23}\Lambda_{34} \quad -B_2\Pi_3\Pi_4\Lambda_{12}\Lambda_{34} \quad +B_2\Pi_3\Pi_4\Lambda_{13}\Lambda_{24}$$

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$$-B_2\Pi_4^2\Lambda_{13}\Lambda_{23} + B_3\Pi_1\Pi_4\Lambda_{23}\Lambda_{24} + B_3\Pi_2\Pi_4\Lambda_{12}\Lambda_{34} - B_3\Pi_2\Pi_4\Lambda_{13}\Lambda_{24}$$
$$-B_3\Pi_4^2\Lambda_{12}\Lambda_{23} - B_4\Pi_1\Pi_4\Lambda_{23}^2 + B_4\Pi_2\Pi_4\Lambda_{13}\Lambda_{23} + B_4\Pi_3\Pi_4\Lambda_{12}\Lambda_{23}\Big\}$$
$$= \frac{m_1}{|J|}\Big\{-2\Pi_1^2\Lambda_{23}\Lambda_{24}\Lambda_{34} + \Pi_1\Pi_3\Lambda_{14}\Lambda_{23}\Lambda_{24} + \Pi_1\Pi_3\Lambda_{12}\Lambda_{24}\Lambda_{34} - \Pi_1\Pi_3\Lambda_{13}\Lambda_{24}^2$$
$$+2\Pi_1\Pi_4\Lambda_{13}\Lambda_{23}\Lambda_{24} + \Pi_1\Pi_4\Lambda_{12}\Lambda_{23}\Lambda_{34} - 2\Pi_1\Pi_4\Lambda_{14}\Lambda_{23}^2 + \Pi_1\Pi_2\Lambda_{13}\Lambda_{34}^2$$
$$+2\Pi_1\Pi_2\Lambda_{14}\Lambda_{23}\Lambda_{34} - 2\Pi_1\Pi_2\Lambda_{12}\Lambda_{34}^2 + \Pi_1\Pi_2\Lambda_{13}\Lambda_{24}\Lambda_{34} - 2\Pi_2^2\Lambda_{13}\Lambda_{14}\Lambda_{34}$$
$$-2\Pi_2\Pi_4\Lambda_{13}^2\Lambda_{24} + 2\Pi_2\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{34} + 2\Pi_2\Pi_4\Lambda_{13}\Lambda_{14}\Lambda_{23} + 2\Pi_2\Pi_3\Lambda_{13}\Lambda_{14}\Lambda_{24}$$
$$+2\Pi_2\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{34} - 2\Pi_2\Pi_3\Lambda_{14}^2\Lambda_{23} - 2\Pi_3\Pi_4\Lambda_{12}^2\Lambda_{34} - \Pi_3^2\Lambda_{12}\Lambda_{13}\Lambda_{24}$$
$$+\Pi_3\Pi_4\Lambda_{12}\Lambda_{14}\Lambda_{24} - \Pi_3^2\Lambda_{12}\Lambda_{14}^2 + 2\Pi_3\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{14} + \Pi_3\Pi_4\Lambda_{12}\Lambda_{14}\Lambda_{23}$$
$$-2\Pi_4^2\Lambda_{12}\Lambda_{13}\Lambda_{23}\Big\} + \frac{\lambda_1}{|J|}\Big\{+B_1\Pi_2^2\Lambda_{34}^2 - 2B_1\Pi_2\Pi_3\Lambda_{24}\Lambda_{34} - 2B_1\Pi_3\Pi_4\Lambda_{23}\Lambda_{24}$$
$$-2B_1\Pi_2\Pi_4\Lambda_{23}\Lambda_{34} - B_2\Pi_1\Pi_2\Lambda_{34}^2 + B_2\Pi_2\Pi_3\Lambda_{14}\Lambda_{34} + B_2\Pi_2\Pi_4\Lambda_{13}\Lambda_{34}$$
$$+B_3\Pi_1\Pi_2\Lambda_{24}\Lambda_{34} - B_3\Pi_2^2\Lambda_{14}\Lambda_{34} - 2B_3\Pi_2\Pi_4\Lambda_{13}\Lambda_{24} + B_3\Pi_2\Pi_4\Lambda_{12}\Lambda_{34}$$
$$+B_3\Pi_2\Pi_4\Lambda_{14}\Lambda_{23} + B_4\Pi_1\Pi_2\Lambda_{23}\Lambda_{34} - B_4\Pi_2^2\Lambda_{13}\Lambda_{34} + B_4\Pi_2\Pi_3\Lambda_{13}\Lambda_{24}$$
$$-2B_4\Pi_2\Pi_3\Lambda_{14}\Lambda_{23} + B_1\Pi_3^2\Lambda_{24}^2 - B_2\Pi_1\Pi_3\Lambda_{24}\Lambda_{34} - B_2\Pi_3^2\Lambda_{14}\Lambda_{24}$$
$$-2B_2\Pi_3\Pi_4\Lambda_{12}\Lambda_{34} + B_2\Pi_3\Pi_4\Lambda_{14}\Lambda_{23} - B_3\Pi_1\Pi_3\Lambda_{24}^2 + B_3\Pi_2\Pi_3\Lambda_{14}\Lambda_{24}$$
$$+B_3\Pi_3\Pi_4\Lambda_{12}\Lambda_{24} + B_4\Pi_1\Pi_3\Lambda_{23}\Lambda_{24} + B_4\Pi_2\Pi_3\Lambda_{12}\Lambda_{34} - B_4\Pi_3^2\Lambda_{12}\Lambda_{24}$$
$$+B_1\Pi_4^2\Lambda_{23}^2 + B_2\Pi_1\Pi_4\Lambda_{23}\Lambda_{34} + B_2\Pi_3\Pi_4\Lambda_{13}\Lambda_{24} - B_2\Pi_4^2\Lambda_{13}\Lambda_{23}$$
$$+B_3\Pi_1\Pi_4\Lambda_{23}\Lambda_{24} - B_3\Pi_4^2\Lambda_{12}\Lambda_{23} - B_4\Pi_1\Pi_4\Lambda_{23}^2 + B_4\Pi_2\Pi_4\Lambda_{13}\Lambda_{23}$$
$$+B_4\Pi_3\Pi_4\Lambda_{12}\Lambda_{23}\Big\}$$
$$= \frac{m_1}{|J|}\Big\{-2\pi_1^2m_1^3m_2m_3m_4 + \pi_1\pi_3m_1^2m_2m_3^2m_4 + \pi_1\pi_3m_1^2m_2m_3^2m_4 - \pi_1\pi_3m_1^2m_2m_3^2m_4$$
$$+\pi_1\pi_4m_1^2m_2m_3m_4^2 + \pi_1\pi_4m_1^2m_2m_3m_4^2 - 2\pi_1\pi_4m_1^2m_2m_3m_4^2 + \pi_1\pi_2m_1^2m_2^3m_4$$
$$+2\pi_1\pi_2m_1^2m_2^2m_3m_4 - 2\pi_1\pi_2m_1^2m_2^2m_3m_4 + \pi_1\pi_2m_1^2m_2^2m_3m_4 - 2\pi_2^2m_1m_2^3m_3m_4$$
$$-2\pi_2\pi_4m_1m_2^2m_3m_4^2 + 2\pi_2\pi_4m_1m_2^2m_3m_4^2 + 2\pi_2\pi_4m_1m_2^2m_3m_4^2 + 2\pi_2\pi_3m_1m_2^2m_3^2m_4$$

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$$+2\pi_2\pi_3 m_1 m_2^2 m_3^2 m_4 \quad -2\pi_2\pi_3 m_1 m_2^2 m_3^2 m_4 \quad -\pi_3^2 m_1 m_2 m_3^2 m_4^2 \quad +\pi_3\pi_4 m_1 m_3^3 m_4$$
$$-\pi_3^2 m_2^2 m_3^3 m_4 \quad +2\pi_3\pi_4 m_2^2 m_3^2 m_4^2 \quad +\pi_3\pi_4 m_2^2 m_3^2 m_4^2 \quad -\pi_4^2 m_1 m_2 m_3 m_4^3\Big\}$$
$$+\frac{\lambda_1}{|J|}\Big\{p_1\pi_2^2 m_1^2 m_2^2 \quad -2p_1\pi_2\pi_3 m_1^2 m_2 m_3 \quad -2p_1\pi_3\pi_4 m_1^2 m_3 m_4 \quad -2p_1\pi_2\pi_4 m_1^2 m_2 m_4$$
$$-p_1\pi_1\pi_2 m_1^2 m_2^2 \quad +p_2\pi_2\pi_3 m_1 m_2^2 m_3 \quad +p_2\pi_2\pi_4 m_1 m_2^2 m_4 \quad +p_3\pi_1\pi_2 m_1^2 m_2 m_3$$
$$-p_3\pi_2^2 m_1 m_2^2 m_3 \quad -2p_3\pi_2\pi_4 m_1 m_2 m_3 m_4 \quad +p_3\pi_2\pi_4 m_1 m_2 m_3 m_4 \quad +p_3\pi_2\pi_4 m_1 m_2 m_3 m_4$$
$$+p_4\pi_2\pi_3 m_1 m_2 m_3 m_4 \quad -p_4\pi_2\pi_3 m_1 m_2 m_3 m_4 \quad +p_1\pi_3^2 m_1^2 m_3^2 \quad -p_2\pi_1\pi_3 m_1^2 m_2 m_3$$
$$-p_2\pi_3^2 m_1 m_2 m_3^2 \quad -2p_2\pi_3\pi_4 m_1 m_2 m_3 m_4 \quad +p_2\pi_3\pi_4 m_1 m_2 m_3 m_4 \quad -p_3\pi_1\pi_3 m_1^2 m_3^2$$
$$+p_3\pi_3\pi_4 m_1 m_3^2 m_4 \quad +p_4\pi_1\pi_3 m_1^2 m_3 m_4 \quad +p_4\pi_2\pi_3 m_1 m_2 m_3 m_4 \quad -p_4\pi_3^2 m_1 m_3^2 m_4$$
$$+p_1\pi_4^2 m_1^2 m_4^2 \quad +p_2\pi_1\pi_4 m_1^2 m_2 m_4 \quad +p_2\pi_3\pi_4 m_1 m_2 m_3 m_4 \quad -p_2\pi_4^2 m_1 m_2 m_4^2$$
$$+p_3\pi_1\pi_4 m_1^2 m_3 m_4 - p_3\pi_4^2 m_1 m_3 m_4^2 - p_4\pi_1\pi_4 m_1^2 m_4^2 + p_4\pi_2\pi_4 m_1 m_2 m_4^2$$
$$+p_4\pi_3\pi_4 m_1 m_3 m_4^2\Big\}. \tag{18}$$

Using $\pi_3 = \pi_1$ and $\pi_4 = \pi_2$; and $p_3 = p_1$ and $p_4 = p_2$, and also $m_1 = m_2 = m_3 = m_4 = 1$ in (18) we get,

$$\frac{\partial\lambda_1}{\partial p_1} = \frac{1}{|J|}\left(-3\pi_1^2 + 8\pi_1\pi_2 - \pi_2^2\right) + \frac{\lambda_1}{|J|}\left(-2p_1\pi_2^2 + 4p_2\pi_1\pi_2 - 2p_1\pi_1\pi_2 - 2p_2\pi_1^2\right). \tag{19}$$

Using $m_1 = m_2 = m_3 = m_4 = 1$ in (9) we get,

$$\lambda_1 = \frac{\pi_2 - \pi_1}{\pi_2 p_1 - \pi_1 p_2} \tag{20}$$

where $\pi_2 p_1 \neq \pi_1 p_2$.

Now using $|J| = |H| = -2p_1 p_2 \pi_1 \pi_2$ from (8) and value of λ_1 from (20) in (19) we get,

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$$\frac{\partial \lambda_1}{\partial p_1} = \frac{1}{2p_1p_2\pi_1\pi_2}\left(3\pi_1^2 - 8\pi_1\pi_2 + \pi_2^2\right)$$

$$+\frac{\pi_2 - \pi_1}{2p_1p_2\pi_1\pi_2(\pi_2p_1 - \pi_1p_2)}\left(2p_1\pi_2^2 - 4p_2\pi_1\pi_2 + 2p_1\pi_1\pi_2 + 2p_2\pi_1^2\right)$$

$$\frac{\partial \lambda_1}{\partial p_1} = \frac{3p_1\pi_2^3 - 5p_2\pi_1\pi_2^2 + p_1\pi_1^2\pi_2 + 14p_2\pi_1^2\pi_2 - 8p_1\pi_1\pi_2^2 - 5p_2\pi_1^3}{2p_1p_2\pi_1\pi_2(\pi_2p_1 - \pi_1p_2)} \quad (21)$$

where $\pi_2 p_1 \neq \pi_1 p_2$.

Now $\pi_1 = \pi_2 = \pi$ in (21) we get,

$$\frac{\partial \lambda_1}{\partial p_1} = -\frac{1}{2p_1p_2} < 0 \quad (22)$$

where $2p_1p_2 > 0$. The relation (22) shows that if the price p_1 of the goods m_1 surges, marginal utility decreases. Therefore, if the price of per unit of the goods m_1 surges \$1.00, the level of purchase of the goods will drop exactly λ_1 units. In this situation, we have realized that the goods m_1 has several substitutes [Mohajan & Mohajan, 2022c].

Now we consult about the behavior of Lagrange multiplier λ_2 when the price p_1 of goods m_1 upsurges. Taking T_{21} from (16) we have [Islam et al., 2010],

$$\frac{\partial \lambda_2}{\partial p_1} = \frac{m_1}{|J|}[C_{12}] + \frac{\lambda_1}{|J|}[C_{32}]$$

$$= \frac{m_1}{|J|}\text{Cofactor of } C_{12} + \frac{\lambda_1}{|J|}\text{Cofactor of } C_{32}$$

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$$= -\frac{m_1}{|J|}\begin{vmatrix} 0 & -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ -B_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -B_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -B_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -B_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix}$$

$$-\frac{\lambda_1}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ -B_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -B_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -B_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix}$$

$$= -\frac{m_1}{|J|}\left\{\Pi_1\begin{vmatrix} -B_1 & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -B_2 & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -B_3 & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -B_4 & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix}\right.$$

$$-\Pi_2\begin{vmatrix} -B_1 & \Lambda_{11} & \Lambda_{13} & \Lambda_{14} \\ -B_2 & \Lambda_{21} & \Lambda_{23} & \Lambda_{24} \\ -B_3 & \Lambda_{31} & \Lambda_{33} & \Lambda_{34} \\ -B_4 & \Lambda_{41} & \Lambda_{43} & \Lambda_{44} \end{vmatrix}$$

$$+\Pi_3\begin{vmatrix} -B_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{14} \\ -B_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{24} \\ -B_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{34} \\ -B_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{44} \end{vmatrix}$$

$$\left.-\Pi_4\begin{vmatrix} -B_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ -B_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ -B_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \\ -B_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} \end{vmatrix}\right\}$$

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$$-\frac{\lambda_1}{|J|}\left\{-B_2\begin{vmatrix}-B_1 & -B_2 & -B_3 & -B_4\\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + B_3\begin{vmatrix}-B_1 & -B_2 & -B_3 & -B_4\\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix}\right.$$

$$\left. - B_4\begin{vmatrix}-B_1 & -B_2 & -B_3 & -B_4\\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34}\end{vmatrix}\right\}$$

$$= -\frac{m_1}{|J|}\left[\Pi_1\left\{-B_1\begin{vmatrix}\Lambda_{22} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{32} & \Lambda_{33} & \Lambda_{34}\\ \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} - \Lambda_{12}\begin{vmatrix}-B_2 & \Lambda_{23} & \Lambda_{24}\\ -B_3 & \Lambda_{33} & \Lambda_{34}\\ -B_4 & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + \Lambda_{13}\begin{vmatrix}-B_2 & \Lambda_{22} & \Lambda_{24}\\ -B_3 & \Lambda_{32} & \Lambda_{34}\\ -B_4 & \Lambda_{42} & \Lambda_{44}\end{vmatrix}\right.\right.$$

$$\left.\left. - \Lambda_{14}\begin{vmatrix}-B_2 & \Lambda_{22} & \Lambda_{23}\\ -B_3 & \Lambda_{32} & \Lambda_{33}\\ -B_4 & \Lambda_{42} & \Lambda_{43}\end{vmatrix}\right\} - \Pi_2\left\{-B_1\begin{vmatrix}\Lambda_{21} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{33} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + \Lambda_{13}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{24}\\ -B_3 & \Lambda_{31} & \Lambda_{34}\\ -B_4 & \Lambda_{41} & \Lambda_{44}\end{vmatrix}\right.$$

$$\left. - \Lambda_{14}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{23}\\ -B_3 & \Lambda_{31} & \Lambda_{33}\\ -B_4 & \Lambda_{41} & \Lambda_{43}\end{vmatrix}\right\} + \Pi_3\left\{-B_1\begin{vmatrix}\Lambda_{21} & \Lambda_{22} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{44}\end{vmatrix} + \Lambda_{12}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{24}\\ -B_3 & \Lambda_{31} & \Lambda_{34}\\ -B_4 & \Lambda_{41} & \Lambda_{44}\end{vmatrix}\right.$$

$$\left. - \Lambda_{14}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{22}\\ -B_3 & \Lambda_{31} & \Lambda_{32}\\ -B_4 & \Lambda_{41} & \Lambda_{42}\end{vmatrix}\right\} - \Pi_4\left\{-B_1\begin{vmatrix}\Lambda_{21} & \Lambda_{22} & \Lambda_{23}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43}\end{vmatrix} + \Lambda_{12}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{23}\\ -B_3 & \Lambda_{31} & \Lambda_{33}\\ -B_4 & \Lambda_{41} & \Lambda_{43}\end{vmatrix}\right.$$

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$$-\Lambda_{13}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{22}\\ -B_3 & \Lambda_{31} & \Lambda_{32}\\ -B_4 & \Lambda_{41} & \Lambda_{42}\end{vmatrix}\Bigg\}\Bigg] \quad -\frac{\lambda_1}{|J|}\Bigg[-B_2\Bigg\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{32} & \Lambda_{33} & \Lambda_{34}\\ \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{31} & \Lambda_{33} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{43} & \Lambda_{44}\end{vmatrix}$$

$$-B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{44}\end{vmatrix}$$

$$+B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43}\end{vmatrix}\Bigg\} \quad +B_3\Bigg\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{21} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{41} & \Lambda_{43} & \Lambda_{44}\end{vmatrix}$$

$$-B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{24}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{44}\end{vmatrix} + B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43}\end{vmatrix}\Bigg\} \quad -B_4\Bigg\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{32} & \Lambda_{33} & \Lambda_{34}\end{vmatrix}$$

$$+B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{21} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{33} & \Lambda_{34}\end{vmatrix} - B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{34}\end{vmatrix} + B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33}\end{vmatrix}\Bigg\}\Bigg]$$

$$= -\frac{m_1}{|J|}\{-B_1\Pi_1\Lambda_{23}\Lambda_{24}\Lambda_{34} \; -B_1\Pi_1\Lambda_{23}\Lambda_{24}\Lambda_{34} \; -B_2\Pi_1\Lambda_{12}\Lambda_{34}^2 \; +B_4\Pi_1\Lambda_{12}\Lambda_{23}\Lambda_{34}$$

$$+B_3\Pi_1\Lambda_{12}\Lambda_{24}\Lambda_{34} \quad +B_2\Pi_1\Lambda_{13}\Lambda_{24}\Lambda_{34} \quad -B_3\Pi_1\Lambda_{13}\Lambda_{24}^2 \quad +B_4\Pi_1\Lambda_{13}\Lambda_{23}\Lambda_{24}$$

$$-B_2\Pi_1\Lambda_{14}\Lambda_{23}\Lambda_{34} \quad +B_3\Pi_1\Lambda_{14}\Lambda_{23}\Lambda_{24} \quad -B_4\Pi_1\Lambda_{14}\Lambda_{23}^2 \quad -B_1\Pi_2\Lambda_{12}\Lambda_{34}^2$$

$$-B_1\Pi_2\Lambda_{14}\Lambda_{23}\Lambda_{34} \quad -B_1\Pi_2\Lambda_{13}\Lambda_{24}\Lambda_{34} \quad -B_2\Pi_2\Lambda_{13}\Lambda_{14}\Lambda_{34} \quad +B_4\Pi_2\Lambda_{12}\Lambda_{13}\Lambda_{34}$$

$$-B_4\Pi_2\Lambda_{13}^2\Lambda_{24} \quad +B_4\Pi_2\Lambda_{13}\Lambda_{14}\Lambda_{24} \quad -B_2\Pi_2\Lambda_{13}\Lambda_{14}\Lambda_{34} \quad +B_3\Pi_2\Lambda_{12}\Lambda_{14}\Lambda_{34}$$

$$-B_3\Pi_2\Lambda_{14}^2\Lambda_{23} \quad +B_4\Pi_2\Lambda_{13}\Lambda_{14}\Lambda_{23} \quad +B_1\Pi_3\Lambda_{12}\Lambda_{24}\Lambda_{34} \quad -B_1\Pi_3\Lambda_{13}\Lambda_{24}^2$$

$$+B_1\Pi_3\Lambda_{14}\Lambda_{23}\Lambda_{24} \quad +B_2\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{34} \quad -B_4\Pi_3\Lambda_{12}^2\Lambda_{34} \quad -B_3\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{24}$$

$$+B_4\Pi_3\Lambda_{12}\Lambda_{13}\Lambda_{24} \quad +B_2\Pi_3\Lambda_{13}\Lambda_{14}\Lambda_{24} \quad -B_2\Pi_3\Lambda_{14}^2\Lambda_{23} \quad -B_3\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{24}$$

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$$+B_4\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{23} \quad +B_1\Pi_4\Lambda_{12}\Lambda_{23}\Lambda_{34} \quad +B_1\Pi_4\Lambda_{13}\Lambda_{23}\Lambda_{24} \quad -B_1\Pi_4\Lambda_{14}\Lambda_{23}^2$$
$$+B_2\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{34} \quad -B_3\Pi_4\Lambda_{12}^2\Lambda_{34} \quad +B_3\Pi_4\Lambda_{12}\Lambda_{14}\Lambda_{23} \quad -B_4\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{23}$$
$$-B_2\Pi_4\Lambda_{13}^2\Lambda_{24} \quad +B_2\Pi_4\Lambda_{13}\Lambda_{14}\Lambda_{23} \quad +B_3\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{24} \quad -B_4\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{23}\Big\}$$

$$-\frac{\lambda_1}{|J|}\Big\{B_1B_2\Pi_2\Lambda_{34}^2 - B_1B_2\Pi_3\Lambda_{24}\Lambda_{34} - B_1B_2\Pi_4\Lambda_{23}\Lambda_{34} - B_2^2\Pi_1\Lambda_{34}^2 + B_2^2\Pi_3\Lambda_{14}\Lambda_{34}$$
$$-B_2^2\Pi_4\Lambda_{13}\Lambda_{34} \quad +B_2B_3\Pi_1\Lambda_{24}\Lambda_{34} \quad -B_2B_3\Pi_2\Lambda_{14}\Lambda_{24} \quad -B_2B_3\Pi_4\Lambda_{13}\Lambda_{24}$$
$$+B_2B_3\Pi_4\Lambda_{14}\Lambda_{23} \quad +B_2B_4\Pi_1\Lambda_{23}\Lambda_{34} \quad -B_2B_4\Pi_2\Lambda_{13}\Lambda_{34} \quad +B_2B_4\Pi_3\Lambda_{13}\Lambda_{24}$$
$$-B_2B_4\Pi_3\Lambda_{14}\Lambda_{23} \quad -B_1B_3\Pi_2\Lambda_{24}\Lambda_{34} \quad +B_1B_3\Pi_3\Lambda_{24}^2 \quad -B_1B_3\Pi_4\Lambda_{23}\Lambda_{24}$$
$$+B_2B\Pi_1\Lambda_{24}\Lambda_{34} \quad -B_2B_3\Pi_2\Lambda_{14}\Lambda_{24} \quad -B_2B_3\Pi_4\Lambda_{12}\Lambda_{34} \quad +B_2B_3\Pi_4\Lambda_{14}\Lambda_{23}$$
$$-B_3^2\Pi_1\Lambda_{24}^2 \quad +B_3^2\Pi_2\Lambda_{14}\Lambda_{24} \quad +B_3^2\Pi_4\Lambda_{12}\Lambda_{24} \quad +B_3B_4\Pi_1\Lambda_{23}\Lambda_{24} \quad +B_3B_4\Pi_2\Lambda_{12}\Lambda_{34}$$
$$-B_3B_4\Pi_2\Lambda_{14}\Lambda_{23} \quad -B_3B_4\Pi_3\Lambda_{12}\Lambda_{24} \quad -B_1B_4\Pi_2\Lambda_{23}\Lambda_{34} \quad -B_1B_4\Pi_3\Lambda_{23}\Lambda_{24}$$
$$+B_1B_4\Pi_4\Lambda_{23}^2 \quad +B_2B_4\Pi_1\Lambda_{23}\Lambda_{34} \quad -B_2B_4\Pi_3\Lambda_{12}\Lambda_{34} \quad +B_2B_4\Pi_3\Lambda_{13}\Lambda_{24}$$
$$-B_2B_4\Pi_4\Lambda_{13}\Lambda_{23} \quad +B_3B_4\Pi_1\Lambda_{23}\Lambda_{24} \quad +B_3B_4\Pi_2\Lambda_{12}\Lambda_{34} \quad -B_3B_4\Pi_2\Lambda_{13}\Lambda_{24}$$
$$-B_3B_4\Pi_4\Lambda_{12}\Lambda_{23} - B_4^2\Pi_1\Lambda_{23}^2 + B_4^2\Pi_2\Lambda_{13}\Lambda_{23} + B_4^2\Pi_3\Lambda_{12}\Lambda_{23}\Big\}$$

$$\frac{\partial\lambda_2}{\partial p_1} = -\frac{m_1}{|J|}\Big\{-2p_1\pi_1m_1^3m_2m_3m_4 \quad -p_2\pi_1m_1^2m_2^2m_3m_4 \quad +p_4\pi_1m_1^2m_2m_3m_4^2$$
$$+p_3\pi_1m_1^2m_2m_3^2m_4 \quad +p_2\pi_1m_1^2m_2^2m_3m_4 \quad -p_3\pi_1m_1^2m_2m_3^2m_4 \quad +p_4\pi_1m_1^2m_2m_3m_4^2$$
$$-p_2\pi_1m_1^2m_2^2m_3m_4 \quad -p_3\pi_1m_1^2m_2m_3^2m_4 \quad -p_4\pi_1m_1^2m_2m_3m_4^2 \quad -p_1\pi_2m_1^2m_2^2m_3m_4$$
$$-p_1\pi_2m_1^2m_2^2m_3m_4 \quad -p_1\pi_2m_1^2m_2^2m_3m_4 \quad -p_2\pi_2m_1m_2^3m_3m_4 \quad +p_4\pi_2m_1m_2^2m_3m_4^2$$
$$-p_4\pi_2m_1m_2^2m_3m_4^2 \quad +p_4\pi_2m_1m_2^2m_3^2m_4 \quad -p_2\pi_2m_1m_2^3m_3m_4 \quad +p_3\pi_2m_1m_2^2m_3^2m_4$$
$$-p_3\pi_2m_1m_2^2m_3^2m_4 \quad +p_4\pi_2m_1m_2^2m_3m_4^2 \quad +p_1\pi_3m_1^2m_2m_3^2m_4 \quad -p_1\pi_3m_1^2m_2m_3^2m_4$$
$$+p_1\pi_3m_1^2m_2m_3^2m_4 \quad -p_2\pi_3m_1m_2^2m_3^2m_4 \quad -p_4\pi_3m_1m_2m_3^2m_4^2 \quad -p_3\pi_3m_1m_2m_3^3m_4$$
$$+p_4\pi_3m_1m_2m_3^2m_4^2 \quad +p_2\pi_3m_1m_2^2m_3^2m_4 \quad -p_2\pi_3m_1m_2^2m_3^2m_4 \quad -p_3\pi_3m_1m_2m_3^3m_4$$
$$+p_4\pi_3m_1m_2m_3^2m_4^2 \quad +p_1\pi_4m_1^2m_2m_3m_4^2 \quad +p_1\pi_4m_1^2m_2m_3m_4^2 \quad -p_1\pi_4m_1^2m_2m_3m_4^2$$
$$+p_2\pi_4m_1m_2^2m_3m_4^2 \quad -p_3\pi_4m_1m_2m_3^2m_4^2 \quad +p_3\pi_4m_1m_2m_3^2m_4^2 \quad -p_4\pi_4m_1m_2m_3m_4^3$$

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$$- p_4\pi_2 m_1 m_2^2 m_3 m_4^2 \quad - p_2\pi_4 m_1 m_2^2 m_3 m_4^2 \quad + p_2\pi_4 m_1 m_2^2 m_3 m_4^2 \quad + p_3\pi_4 m_1 m_2 m_3^2 m_4^2$$
$$- p_4\pi_4 m_1 m_2 m_3 m_4^2\Big\} \quad -\frac{\lambda_1}{|J|}\Big\{p_1 p_2\pi_2 m_1^2 m_2^2 \quad - p_1 p_2\pi_3 m_1^2 m_2 m_3 \quad - p_1 p_2\pi_4 m_1^2 m_2 m_4$$
$$- p_2^2\pi_1 m_1^2 m_2^2 \quad + p_2^2\pi_3 m_1 m_2^2 m_3 \quad - p_2^2\pi_4 m_1 m_2^2 m_4 \quad + p_2 p_3\pi_1 m_1^2 m_2 m_3$$
$$- p_2 p_3\pi_2 m_1 m_2 m_3^2 \quad - p_2 p_3\pi_4 m_1 m_2 m_3 m_4 \quad + p_2 p_3\pi_4 m_1 m_2 m_3 m_4 \quad + p_2 p_4\pi_1 m_1^2 m_2 m_4$$
$$- p_2 p_4\pi_2 m_1 m_2^2 m_4 \quad + p_2 p_4\pi_3 m_1 m_2 m_3 m_4 \quad - p_2 p_4\pi_3 m_1 m_2 m_3 m_4 \quad - p_2 p_3\pi_2 m_1^2 m_2 m_3$$
$$- p_1 p_3\pi_3 m_1^2 m_3^2 \quad - p_1 p_3\pi_4 m_1^2 m_3 m_4 \quad + p_2 p_3\pi_1 m_1^2 m_2 m_3 \quad - p_2 p_3\pi_2 m_1 m_2 m_3^2$$
$$- p_2 p_3\pi_4 m_1 m_2 m_3 m_4 \quad + p_2 p_3\pi_4 m_1 m_2 m_3 m_4 \quad - p_3^2\pi_1 m_1^2 m_3^2 \quad + p_3^2\pi_2 m_1 m_2 m_3^2$$
$$+ p_3 p_4\pi_1 m_1^2 m_3 m_4 \quad + p_3 p_4\pi_2 m_1 m_2 m_3 m_4 \quad - p_3 p_4\pi_2 m_1 m_2 m_3 m_4 \quad - p_3 p_4\pi_3 m_1 m_3^2 m_4$$
$$- p_1 p_4\pi_2 m_1^2 m_2 m_4 \quad - p_1 p_4\pi_3 m_1^2 m_3 m_4 \quad + p_1 p_4\pi_4 m_1^2 m_4^2 \quad + p_2 p_4\pi_1 m_1^2 m_2 m_4$$
$$- p_2 p_4\pi_3 m_1 m_2 m_3 m_4 \quad + p_2 p_4\pi_3 m_1 m_2 m_3 m_4 \quad - p_2 p_4\pi_4 m_1 m_2 m_4^2 \quad + p_3 p_4\pi_1 m_1^2 m_3 m_4$$
$$+ p_2 p_4\pi_3 m_1 m_2 m_3 m_4 + p_3 p_4\pi_2 m_1 m_2 m_3 m_4 - p_3 p_4\pi_2 m_1 m_2 m_3 m_4 - p_3 p_4\pi_4 m_1 m_3 m_4^2$$
$$- p_4^2\pi_1 m_1^2 m_4^2 + p_4^2\pi_2 m_1 m_2 m_4^2$$
$$+ p_4^2\pi_3 m_1 m_3 m_4^2\Big\}. \tag{23}$$

Using $\pi_3 = \pi_1$ and $\pi_4 = \pi_2$, $p_3 = p_1$ and $p_4 = p_2$, and also $m_3 = m_1$ and $m_4 = m_2$ in (23) we get,

$$\frac{\partial\lambda_2}{\partial p_1} = -\frac{m_1}{|J|}\Big\{-4 p_1\pi_1 m_1^4 m_2^2 \quad - p_1\pi_2 m_1^3 m_2^3 \quad - 2 p_2\pi_2 m_1^2 m_2^4 \quad + p_2\pi_2 m_1^4 m_2^2$$
$$- p_2\pi_2 m_1^3 m_2^3\Big\} \quad -\frac{\lambda_1}{|J|}\Big\{-2 p_1 p_2\pi_2 m_1^2 m_2^2 \quad - 2 p_2^2\pi_2 m_1 m_2^3 \quad + 3 p_2^2\pi_1 m_1^2 m_2^2$$
$$- p_1 p_2\pi_2 m_1^3 m_2 - 3 p_1^2\pi_1 m_1^4 + p_1^2\pi_2 m_1^3 m_2\Big\}. \tag{24}$$

Using $m_1 = m_2 = m_3 = m_4 = 1$ in (9) we get,

$$\lambda_1 = \frac{\pi_2 - \pi_1}{\pi_2 p_1 - \pi_1 p_2} \tag{25}$$

where $\pi_2 p_1 \neq \pi_1 p_2$.

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Now using $|J| = |H| = -2p_1p_2\pi_1\pi_2$ from (8), $m_1 = m_2 = 1$ and value of λ_1 from (25) in (24) we get,

$$\frac{\partial \lambda_2}{\partial p_1} = -\frac{2p_2\pi_2 + p_1\pi_2 + 4p_1\pi_1}{2p_1p_2\pi_1\pi_2} + \frac{(\pi_2 - \pi_1)(3p_2^2\pi_1 - 3p_1p_2\pi_2 - 2p_2^2\pi_2 - 3p_1^2\pi_1 + p_1^2\pi_2)}{2p_1p_2\pi_1\pi_2(\pi_2p_1 - \pi_1p_2)}$$

$$\frac{\partial \lambda_2}{\partial p_1} = \frac{2p_1^2\pi_2^2 - 3p_2^2\pi_1^2 - 2p_2^2\pi_2^2 - 7p_1^2\pi_1\pi_2 + 4p_1p_2\pi_1^2 + 7p_2^2\pi_1\pi_2 - 4p_1p_2\pi_2^2 + 3p_1^2\pi_1^2 + 3p_1p_2\pi_1\pi_2}{2p_1p_2\pi_1\pi_2(\pi_2p_1 - \pi_1p_2)} \quad (26)$$

Now using $\pi_1 = \pi_2 = \pi$ in (26) we get,

$$\frac{\partial \lambda_2}{\partial p_1} = -\frac{(p_1 - 2p_2)(2p_1 + p_2)}{2p_1p_2\pi(p_1 - p_2)} \quad (27)$$

where $2p_1p_2\pi > 0$.

Now if $p_1 > 2p_2$ in (27) we observe that,

$$\frac{\partial \lambda_2}{\partial p_1} < 0. \quad (28)$$

The relation (28) we have seen that if the price p_1 of the good m_1 goes up, marginal utility goes down. Therefore, if the price of per unit of product m_1 raises \$1.00, the level of buying will decline exactly λ_2 units. Therefore, product m_1 has numerous substitutes; and customers move to substitutes when price of good m_1 goes up [Islam et al., 2010].

Now if $p_2 < p_1 < 2p_2$ in (27) we observe that,

$$\frac{\partial \lambda_2}{\partial p_1} > 0. \quad (29)$$

The inequality (29) shows that if the price p_1 of the goods m_1 surges, marginal utility also surges. Therefore, if the price of per unit of goods m_1 raises \$1.00, the level of buying will raise exactly λ_2 units. Therefore, goods m_1 is a best quality

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product, and it has no substitutes [Mohajan & Mohajan, 2023b]. The firm may try for more production of the commodity m_1 to obtain maximum profit, if $p_2 < p_1 < 2p_2$, where p_2 is the price of the commodity m_2.

We examine the properties of λ_1 when unit price p_4 of commodity m_4 upsurges. We take T_{14} from (17) we can write [Islam et al., 2011; Mohajan & Mohajan, 2022a],

$$\frac{\partial \lambda_1}{\partial p_4} = \frac{m_4}{|J|}[C_{11}] + \frac{\lambda_1}{|J|}[C_{61}]$$

$$= \frac{m_4}{|J|}\text{Cofactor of } C_{11} + \frac{\lambda_1}{|J|}\text{Cofactor of } C_{61}$$

$$= \frac{m_4}{|J|}\begin{vmatrix} 0 & -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ -\Pi_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -\Pi_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix}$$

$$-\frac{\lambda_1}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ -\Pi_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -\Pi_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \end{vmatrix}$$

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$$= \frac{m_4}{|J|}\left\{\Pi_1\begin{vmatrix} -\Pi_1 & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -\Pi_2 & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix} - \Pi_2\begin{vmatrix} -\Pi_1 & \Lambda_{11} & \Lambda_{13} & \Lambda_{14} \\ -\Pi_2 & \Lambda_{21} & \Lambda_{23} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{31} & \Lambda_{33} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{41} & \Lambda_{43} & \Lambda_{44} \end{vmatrix}\right.$$

$$\left. + \Pi_3\begin{vmatrix} -\Pi_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{14} \\ -\Pi_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{44} \end{vmatrix} - \Pi_4\begin{vmatrix} -\Pi_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ -\Pi_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ -\Pi_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \\ -\Pi_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} \end{vmatrix}\right\}$$

$$- \frac{\lambda_1}{|J|}\left\{-\Pi_1\begin{vmatrix} -B_1 & -B_2 & -B_3 & -B_4 \\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \end{vmatrix} + \Pi_2\begin{vmatrix} -B_1 & -B_2 & -B_3 & -B_4 \\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \end{vmatrix}\right.$$

$$\left. - \Pi_3\begin{vmatrix} -B_1 & -B_2 & -B_3 & -B_4 \\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \end{vmatrix}\right\}$$

$$= \frac{m_4}{|J|}\left[\Pi_1\left\{-\Pi_1\begin{vmatrix} \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix} - \Lambda_{12}\begin{vmatrix} -\Pi_2 & \Lambda_{23} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{33} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{43} & \Lambda_{44} \end{vmatrix} + \Lambda_{13}\begin{vmatrix} -\Pi_2 & \Lambda_{22} & \Lambda_{24} \\ -\Pi_3 & \Lambda_{32} & \Lambda_{34} \\ -\Pi_4 & \Lambda_{42} & \Lambda_{44} \end{vmatrix}\right.\right.$$

$$\left.\left. - \Lambda_{14}\begin{vmatrix} -\Pi_2 & \Lambda_{22} & \Lambda_{23} \\ -\Pi_3 & \Lambda_{32} & \Lambda_{33} \\ -\Pi_4 & \Lambda_{42} & \Lambda_{43} \end{vmatrix}\right\}\right.$$

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$$-\Pi_2\left\{-\Pi_1\begin{vmatrix}\Lambda_{21} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{33} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + \Lambda_{13}\begin{vmatrix}-\Pi_2 & \Lambda_{21} & \Lambda_{24}\\ -\Pi_3 & \Lambda_{31} & \Lambda_{34}\\ -\Pi_4 & \Lambda_{41} & \Lambda_{44}\end{vmatrix} - \Lambda_{14}\begin{vmatrix}-\Pi_2 & \Lambda_{21} & \Lambda_{23}\\ -\Pi_3 & \Lambda_{31} & \Lambda_{33}\\ -\Pi_4 & \Lambda_{41} & \Lambda_{43}\end{vmatrix}\right\}$$

$$+\Pi_3\left\{-\Pi_1\begin{vmatrix}\Lambda_{21} & \Lambda_{22} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{44}\end{vmatrix} + \Lambda_{12}\begin{vmatrix}-\Pi_2 & \Lambda_{21} & \Lambda_{24}\\ -\Pi_3 & \Lambda_{31} & \Lambda_{34}\\ -\Pi_4 & \Lambda_{41} & \Lambda_{44}\end{vmatrix} - \Lambda_{14}\begin{vmatrix}-\Pi_2 & \Lambda_{21} & \Lambda_{22}\\ -\Pi_3 & \Lambda_{31} & \Lambda_{32}\\ -\Pi_4 & \Lambda_{41} & \Lambda_{42}\end{vmatrix}\right\}$$

$$-\Pi_4\left\{-\Pi_1\begin{vmatrix}\Lambda_{21} & \Lambda_{22} & \Lambda_{23}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43}\end{vmatrix} + \Lambda_{12}\begin{vmatrix}-\Pi_2 & \Lambda_{21} & \Lambda_{23}\\ -\Pi_3 & \Lambda_{31} & \Lambda_{33}\\ -\Pi_4 & \Lambda_{41} & \Lambda_{43}\end{vmatrix} - \Lambda_{13}\begin{vmatrix}-\Pi_2 & \Lambda_{21} & \Lambda_{22}\\ -\Pi_3 & \Lambda_{31} & \Lambda_{32}\\ -\Pi_4 & \Lambda_{41} & \Lambda_{42}\end{vmatrix}\right\}\Bigg]$$

$$-\frac{\lambda_1}{|J|}\Bigg[-\Pi_1\left\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{32} & \Lambda_{33} & \Lambda_{34}\end{vmatrix} + B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{21} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{33} & \Lambda_{34}\end{vmatrix} - B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{34}\end{vmatrix}\right.$$

$$\left. + B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33}\end{vmatrix}\right\} + \Pi_2\left\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{12} & \Lambda_{13} & \Lambda_{14}\\ \Lambda_{32} & \Lambda_{33} & \Lambda_{34}\end{vmatrix} + B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{11} & \Lambda_{13} & \Lambda_{14}\\ \Lambda_{31} & \Lambda_{33} & \Lambda_{34}\end{vmatrix}\right.$$

$$\left. - B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{11} & \Lambda_{12} & \Lambda_{14}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{34}\end{vmatrix} + B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{11} & \Lambda_{12} & \Lambda_{13}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33}\end{vmatrix}\right\} - \Pi_3\left\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{12} & \Lambda_{13} & \Lambda_{14}\\ \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\end{vmatrix}\right.$$

$$\left. + B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{11} & \Lambda_{13} & \Lambda_{14}\\ \Lambda_{21} & \Lambda_{23} & \Lambda_{24}\end{vmatrix} - B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{11} & \Lambda_{12} & \Lambda_{14}\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{24}\end{vmatrix} + B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{11} & \Lambda_{12} & \Lambda_{13}\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23}\end{vmatrix}\right\}\Bigg]$$

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$$= \frac{m_4}{|J|}\{-\Pi_1^2\Lambda_{23}\Lambda_{24}\Lambda_{34} \quad -\Pi_1^2\Lambda_{23}\Lambda_{24}\Lambda_{34} \quad -\Pi_1\Pi_2\Lambda_{12}\Lambda_{34}^2 \quad -\Pi_1\Pi_4\Lambda_{12}\Lambda_{23}\Lambda_{34}$$
$$+\Pi_1\Pi_3\Lambda_{12}\Lambda_{24}\Lambda_{34} \quad +\Pi_1\Pi_4\Lambda_{12}\Lambda_{23}\Lambda_{34} \quad +\Pi_1\Pi_2\Lambda_{13}\Lambda_{34}^2 \quad -\Pi_1\Pi_3\Lambda_{13}\Lambda_{24}^2$$
$$+\Pi_1\Pi_4\Lambda_{13}\Lambda_{23}\Lambda_{24} \quad +\Pi_1\Pi_2\Lambda_{14}\Lambda_{23}\Lambda_{34} \quad +\Pi_1\Pi_3\Lambda_{14}\Lambda_{23}\Lambda_{24} \quad -\Pi_1\Pi_4\Lambda_{14}\Lambda_{23}^2$$
$$-\Pi_1\Pi_2\Lambda_{12}\Lambda_{34}^2 \quad +\Pi_1\Pi_2\Lambda_{14}\Lambda_{23}\Lambda_{34} \quad +\Pi_1\Pi_2\Lambda_{13}\Lambda_{24}\Lambda_{34} \quad -\Pi_2^2\Lambda_{13}\Lambda_{14}\Lambda_{34}$$
$$+\Pi_2\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{34} \quad +\Pi_2\Pi_3\Lambda_{13}\Lambda_{14}\Lambda_{24} \quad -\Pi_2\Pi_4\Lambda_{13}^2\Lambda_{24} \quad -\Pi_2^2\Lambda_{13}\Lambda_{14}\Lambda_{34}$$
$$+\Pi_2\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{34} \quad -\Pi_2\Pi_3\Lambda_{14}^2\Lambda_{23} \quad +\Pi_2\Pi_4\Lambda_{13}\Lambda_{14}\Lambda_{23} \quad +\Pi_2\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{34}$$
$$-\Pi_3\Pi_4\Lambda_{12}^2\Lambda_{34} \quad -\Pi_3^2\Lambda_{12}\Lambda_{13}\Lambda_{24} \quad +\Pi_3\Pi_4\Lambda_{12}\Lambda_{14}\Lambda_{24} \quad +\Pi_2\Pi_3\Lambda_{13}\Lambda_{14}\Lambda_{24}$$
$$-\Pi_2\Pi_3\Lambda_{14}^2\Lambda_{23} \quad -\Pi_3^2\Lambda_{12}\Lambda_{14}^2 \quad +\Pi_3\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{14} \quad +\Pi_1\Pi_4\Lambda_{12}\Lambda_{23}\Lambda_{34}$$
$$+\Pi_1\Pi_4\Lambda_{13}\Lambda_{23}\Lambda_{24} \quad -\Pi_1\Pi_4\Lambda_{14}\Lambda_{23}^2 \quad +\Pi_2\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{34} \quad -\Pi_3\Pi_4\Lambda_{12}^2\Lambda_{34}$$
$$+\Pi_3\Pi_4\Lambda_{12}\Lambda_{14}\Lambda_{23} \quad -\Pi_4^2\Lambda_{12}\Lambda_{13}\Lambda_{23} \quad -\Pi_2\Pi_4\Lambda_{13}^2\Lambda_{24} \quad +\Pi_2\Pi_4\Lambda_{13}\Lambda_{14}\Lambda_{23}$$
$$+\Pi_3\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{24} \quad -\Pi_4^2\Lambda_{12}\Lambda_{13}\Lambda_{23}\} \quad -\frac{\lambda_1}{|J|}\{B_1\Pi_1\Pi_2\Lambda_{23}\Lambda_{34} \quad -B_1\Pi_1\Pi_3\Lambda_{23}\Lambda_{24}$$
$$+B_1\Pi_1\Pi_4\Lambda_{23}^2 \quad +B_2\Pi_1^2\Lambda_{23}\Lambda_{34} \quad -B_2\Pi_1\Pi_3\Lambda_{12}\Lambda_{34} \quad +B_2\Pi_1\Pi_3\Lambda_{13}\Lambda_{24}$$
$$-B_2\Pi_1\Pi_4\Lambda_{13}\Lambda_{23} \quad +B_3\Pi_1^2\Lambda_{23}\Lambda_{24} \quad +B_3\Pi_1\Pi_2\Lambda_{12}\Lambda_{34} \quad -B_3\Pi_1\Pi_2\Lambda_{13}\Lambda_{24}$$
$$-B_3\Pi_1\Pi_4\Lambda_{12}\Lambda_{23} - B_4\Pi_1^2\Lambda_{23}^2 + B_4\Pi_1\Pi_2\Lambda_{13}\Lambda_{23} + B_4\Pi_1\Pi_3\Lambda_{12}\Lambda_{23} + B_1\Pi_2^2\Lambda_{13}\Lambda_{34}$$
$$-B_1\Pi_2\Pi_3\Lambda_{12}\Lambda_{34} \quad +B_1\Pi_2\Pi_3\Lambda_{14}\Lambda_{23} \quad -B_1\Pi_2\Pi_4\Lambda_{13}\Lambda_{23} \quad -B_2\Pi_1\Pi_2\Lambda_{13}\Lambda_{34}$$
$$-B_2\Pi_2\Pi_3\Lambda_{13}\Lambda_{14} \quad +B_2\Pi_2\Pi_4\Lambda_{13}^2 \quad +B_3\Pi_1\Pi_2\Lambda_{12}\Lambda_{34} \quad -B_3\Pi_1\Pi_2\Lambda_{14}\Lambda_{23}$$
$$+B_3\Pi_2^2\Lambda_{13}\Lambda_{14} - B_3\Pi_2\Pi_4\Lambda_{12}\Lambda_{13} + B_4\Pi_1\Pi_2\Lambda_{13}\Lambda_{23} - B_4\Pi_2^2\Lambda_{13}^2 + B_4\Pi_2\Pi_3\Lambda_{12}\Lambda_{13}$$
$$-B_1\Pi_2\Pi_3\Lambda_{13}\Lambda_{24} \quad +B_1\Pi_2\Pi_3\Lambda_{14}\Lambda_{23} \quad +B_1\Pi_3^2\Lambda_{12}\Lambda_{24} \quad -B_1\Pi_3\Pi_4\Lambda_{12}\Lambda_{23}$$
$$+B_2\Pi_1\Pi_3\Lambda_{13}\Lambda_{24} \quad -B_2\Pi_1\Pi_3\Lambda_{14}\Lambda_{23} \quad +B_2\Pi_3^2\Lambda_{12}\Lambda_{14} \quad -B_2\Pi_3\Pi_4\Lambda_{12}\Lambda_{13}$$
$$-B_3\Pi_1\Pi_2\Lambda_{12}\Lambda_{24} \quad -B_3\Pi_2\Pi_3\Lambda_{12}\Lambda_{14} \quad +B_3\Pi_3\Pi_4\Lambda_{12}^2 \quad +B_4\Pi_1\Pi_3\Lambda_{12}\Lambda_{23}$$
$$+B_4\Pi_2\Pi_3\Lambda_{12}\Lambda_{13} - B_4\Pi_3^2\Lambda_{12}^2$$

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$$= \frac{m_1}{|J|}\{-2\Pi_1^2\Lambda_{23}\Lambda_{24}\Lambda_{34} + \Pi_1\Pi_3\Lambda_{14}\Lambda_{23}\Lambda_{24} + \Pi_1\Pi_3\Lambda_{12}\Lambda_{24}\Lambda_{34} - \Pi_1\Pi_3\Lambda_{13}\Lambda_{24}^2$$
$$+ 2\Pi_1\Pi_4\Lambda_{13}\Lambda_{23}\Lambda_{24} + \Pi_1\Pi_4\Lambda_{12}\Lambda_{23}\Lambda_{34} - 2\Pi_1\Pi_4\Lambda_{14}\Lambda_{23}^2 + \Pi_1\Pi_2\Lambda_{13}\Lambda_{34}^2$$
$$+ 2\Pi_1\Pi_2\Lambda_{14}\Lambda_{23}\Lambda_{34} - 2\Pi_1\Pi_2\Lambda_{12}\Lambda_{34}^2 + \Pi_1\Pi_2\Lambda_{13}\Lambda_{24}\Lambda_{34} - 2\Pi_2^2\Lambda_{13}\Lambda_{14}\Lambda_{34}$$
$$- 2\Pi_2\Pi_4\Lambda_{13}^2\Lambda_{24} + 2\Pi_2\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{34} + 2\Pi_2\Pi_4 L_{13}L_{14}L_{23} + 2\Pi_2\Pi_3\Lambda_{13}\Lambda_{14}\Lambda_{24}$$
$$+ 2\Pi_2\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{34} - 2\Pi_2\Pi_3\Lambda_{14}^2\Lambda_{23} - 2\Pi_3\Pi_4\Lambda_{12}^2\Lambda_{34} - \Pi_3^2\Lambda_{12}\Lambda_{13}\Lambda_{24}$$
$$+ \Pi_3\Pi_4\Lambda_{12}\Lambda_{14}\Lambda_{24} - \Pi_3^2\Lambda_{12}\Lambda_{14}^2 + 2\Pi_3\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{14} + \Pi_3\Pi_4\Lambda_{12}\Lambda_{14}\Lambda_{23}$$
$$- 2\Pi_4^2\Lambda_{12}\Lambda_{13}\Lambda_{23}\} - \frac{\lambda_1}{|J|}\{B_1\Pi_1\Pi_2\Lambda_{23}\Lambda_{34} - B_1\Pi_1\Pi_3\Lambda_{23}\Lambda_{24} + B_1\Pi_1\Pi_4\Lambda_{23}^2$$
$$+ B_2\Pi_1^2\Lambda_{23}\Lambda_{34} - B_2\Pi_1\Pi_3\Lambda_{12}\Lambda_{34} + 2B_2\Pi_1\Pi_3\Lambda_{13}\Lambda_{24} - B_2\Pi_1\Pi_3\Lambda_{14}\Lambda_{23}$$
$$- B_2\Pi_1\Pi_4\Lambda_{13}\Lambda_{23} + B_3\Pi_1^2\Lambda_{23}\Lambda_{24} + B_3\Pi_1\Pi_2\Lambda_{12}\Lambda_{34} - B_3\Pi_1\Pi_2\Lambda_{13}\Lambda_{24}$$
$$- B_3\Pi_1\Pi_2\Lambda_{14}\Lambda_{23} - B_3\Pi_1\Pi_2\Lambda_{12}\Lambda_{24} - B_3\Pi_1\Pi_4\Lambda_{12}\Lambda_{23} - B_4\Pi_1^2\Lambda_{23}^2$$
$$+ 2B_4\Pi_1\Pi_2\Lambda_{13}\Lambda_{23} + 2B_4\Pi_1\Pi_3\Lambda_{12}\Lambda_{23} + B_1\Pi_2^2\Lambda_{13}\Lambda_{34} - B_1\Pi_2\Pi_3\Lambda_{12}\Lambda_{34}$$
$$+ 2B_1\Pi_2\Pi_3\Lambda_{14}\Lambda_{23} - B_1\Pi_2\Pi_3\Lambda_{13}\Lambda_{24} - B_1\Pi_2\Pi_4\Lambda_{13}\Lambda_{23} - B_2\Pi_1\Pi_2\Lambda_{13}\Lambda_{34}$$
$$- B_2\Pi_2\Pi_3\Lambda_{13}\Lambda_{14} + B_2\Pi_2\Pi_4\Lambda_{13}^2 + B_3\Pi_1\Pi_2\Lambda_{12}\Lambda_{34} + B_3\Pi_2^2\Lambda_{13}\Lambda_{14}$$
$$- B_3\Pi_2\Pi_4\Lambda_{12}\Lambda_{13} - B_4\Pi_2^2\Lambda_{13}^2 + 2B_4\Pi_2\Pi_3\Lambda_{12}\Lambda_{13} + B_1\Pi_3^2\Lambda_{12}\Lambda_{24}$$
$$- B_1\Pi_3\Pi_4\Lambda_{12}\Lambda_{23} + B_2\Pi_3^2\Lambda_{12}\Lambda_{14} - B_2\Pi_3\Pi_4\Lambda_{12}\Lambda_{13} - B_3\Pi_2\Pi_3\Lambda_{12}\Lambda_{14}$$
$$+ B_3\Pi_3\Pi_4\Lambda_{12}^2 - B_4\Pi_3^2\Lambda_{12}^2\}$$

$$= \frac{m_1}{|J|}\{-2\pi_1^2 m_1^3 m_2 m_3 m_4 + \pi_1\pi_3 m_1^2 m_2 m_3^2 m_4 + \pi_1\pi_3 m_1^2 m_2 m_3^2 m_4 - \pi_1\pi_3 m_1^2 m_2 m_3^2 m_4$$
$$+ \pi_1\pi_4 m_1^2 m_2 m_3 m_4^2 + \pi_1\pi_4 m_1^2 m_2 m_3 m_4^2 - 2\pi_1\pi_4 m_1^2 m_2 m_3 m_4^2 + \pi_1\pi_2 m_1^2 m_2^3 m_4$$
$$+ 2\pi_1\pi_2 m_1^2 m_2^2 m_3 m_4 - 2\pi_1\pi_2 m_1^2 m_2^2 m_3 m_4 + \pi_1\pi_2 m_1^2 m_2^2 m_3 m_4 - 2\pi_2^2 m_1 m_2^3 m_3 m_4$$
$$- 2\pi_2\pi_4 m_1 m_2^2 m_3 m_4^2 + 2\pi_2\pi_4 m_1 m_2^2 m_3 m_4^2 + 2\pi_2\pi_4 m_1 m_2^2 m_3 m_4^2 + 2\pi_2\pi_3 m_1 m_2^2 m_3^2 m_4$$
$$+ 2\pi_2\pi_3 m_1 m_2^2 m_3^2 m_4 - 2\pi_2\pi_3 m_1 m_2^2 m_3^2 m_4 - \pi_3^2 m_1 m_2 m_3^2 m_4^2 + \pi_3\pi_4 m_1 m_2 m_3^3 m_4$$
$$- \pi_3^2 m_2^2 m_3^3 m_4 + 2\pi_3\pi_4 m_2^2 m_3^2 m_4^2 + \pi_3\pi_4 m_2^2 m_3^2 m_4^2 - \pi_4^2 m_1 m_2 m_3 m_4^3\}$$

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$$-\frac{\lambda_1}{|J|}\{p_1\pi_1\pi_2 m_1^2 m_2 m_4 - p_1\pi_1\pi_3 m_1^2 m_3 m_4 + p_1\pi_1\pi_4 m_1^2 m_4^2 + p_2\pi_1^2 m_1^2 m_2 m_4$$
$$- p_2\pi_1\pi_3 m_1 m_2 m_3 m_4 + 2p_2\pi_1\pi_3 m_1 m_2 m_3 m_4 - p_2\pi_1\pi_3 m_1 m_2 m_3 m_4$$
$$- p_2\pi_1\pi_4 m_1 m_2 m_4^2 + p_3\pi_1^2 m_1^2 m_3 m_4 + p_3\pi_1\pi_2 m_1 m_2 m_3 m_4 - p_3\pi_1\pi_2 m_1 m_2 m_3 m_4$$
$$- p_3\pi_1\pi_2 m_1 m_2 m_3 m_4 - p_3\pi_1\pi_2 m_1 m_3^2 m_4 - p_3\pi_1\pi_4 m_1 m_3 m_4^2 - p_4\pi_1^2 m_1^2 m_4^2$$
$$+ 2p_4\pi_1\pi_2 m_1 m_2 m_4^2 + 2p_4\pi_1\pi_3 m_1 m_3 m_4^2 + p_1\pi_2^2 m_1 m_2^2 m_4 - p_1\pi_2\pi_3 m_1 m_2 m_3 m_4$$
$$+ 2p_1\pi_2\pi_3 m_1 m_2 m_3 m_4 - p_1\pi_2\pi_3 m_1 m_2 m_3 m_4 - p_1\pi_2\pi_4 m_1 m_2 m_4^2 - p_2\pi_1\pi_2 m_1 m_2^2 m_4$$
$$- p_2\pi_2\pi_3 m_2^2 m_3 m_4 + p_2\pi_2\pi_4 m_2^2 m_4^2 + p_3\pi_1\pi_2 m_1 m_2 m_3 m_4 + p_3\pi_2^2 m_2^2 m_3 m_4$$
$$- p_3\pi_2\pi_4 m_2 m_3 m_4^2 + p_1\pi_3^2 m_1 m_3^2 m_4 + 2p_4\pi_2\pi_3 m_2 m_3 m_4^2 + p_1\pi_3^2 m_1 m_3^2 m_4$$
$$- p_1\pi_3\pi_4 m_1 m_3 m_4^2 + p_2\pi_3^2 m_2 m_3^2 m_4 - p_2\pi_3\pi_4 m_2 m_3 m_4^2 - p_3\pi_2\pi_3 m_2 m_3^2 m_4$$
$$+ p_3\pi_3\pi_4 m_3^2 m_4^2 - p_4\pi_3^2 m_3^2 m_4^2\}$$
$$\frac{\partial\lambda_1}{\partial p_4} = \frac{m_1}{|J|}\{-2\pi_1^2 m_1^3 m_2 m_3 m_4 + \pi_1\pi_3 m_1^2 m_2 m_3^2 m_4 + 2\pi_2\pi_4 m_1 m_2^2 m_3 m_4^2$$
$$+ 2\pi_2\pi_3 m_1 m_2^2 m_3^2 m_4 - \pi_3^2 m_1 m_2 m_3^2 m_4^2 + \pi_3\pi_4 m_1 m_2 m_3^3 m_4 - \pi_3^2 m_2^2 m_3^3 m_4$$
$$+ 3\pi_3\pi_4 m_2^2 m_3^2 m_4^2 - \pi_4^2 m_1 m_2 m_3 m_4^3\} - \frac{\lambda_1}{|J|}\{p_1\pi_1\pi_2 m_1^2 m_2 m_4 - p_1\pi_1\pi_3 m_1^2 m_3 m_4$$
$$+ p_1\pi_1\pi_4 m_1^2 m_4^2 + p_2\pi_1^2 m_1^2 m_2 m_4 - p_2\pi_1\pi_4 m_1 m_2 m_4^2 + p_3\pi_1^2 m_1^2 m_3 m_4$$
$$- p_3\pi_1\pi_2 m_1 m_3^2 m_4 - p_3\pi_1\pi_4 m_1 m_3 m_4^2 - p_4\pi_1^2 m_1^2 m_4^2 + 2p_4\pi_1\pi_2 m_1 m_2 m_4^2$$
$$+ 2p_4\pi_1\pi_3 m_1 m_3 m_4^2 + p_1\pi_2^2 m_1 m_2^2 m_4 - p_1\pi_2\pi_4 m_1 m_2 m_4^2 - p_2\pi_1\pi_2 m_1 m_2^2 m_4$$
$$- p_2\pi_2\pi_3 m_2^2 m_3 m_4 + p_2\pi_2\pi_4 m_2^2 m_4^2 + p_3\pi_2^2 m_2^2 m_3 m_4 - p_3\pi_2\pi_4 m_2 m_3 m_4^2$$
$$+ 2p_1\pi_3^2 m_1 m_3^2 m_4 + 2p_4\pi_2\pi_3 m_2 m_3 m_4^2 - p_1\pi_3\pi_4 m_1 m_3 m_4^2 + p_2\pi_3^2 m_2 m_3^2 m_4$$
$$- p_2\pi_3\pi_4 m_2 m_3 m_4^2 - p_3\pi_2\pi_3 m_2 m_3^2 m_4 + p_3\pi_3\pi_4 m_3^2 m_4^2$$
$$- p_4\pi_3^2 m_3^2 m_4^2\}. \qquad (30)$$

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Using $\pi_3 = \pi_1$ and $\pi_4 = \pi_2$; and $p_3 = p_1$ and $p_4 = p_2$, and also $m_1 = m_2 = m_3 = m_4 = 1$ in (30) we get,

$$\frac{\partial \lambda_1}{\partial p_4} = \frac{1}{|J|}\left(-3\pi_1^2 + 6\pi_1\pi_2 + \pi_2^2\right) - \frac{\lambda_1}{|J|}\left(2p_2\pi_1^2 - p_1\pi_1\pi_2 + p_2\pi_2^2 + 2p_1\pi_1^2\right). \tag{31}$$

Now using $|J| = |H| = -2p_1p_2\pi_1\pi_2$ from (8), and λ_1 from (9) in (31) we get,

$$\frac{\partial \lambda_1}{\partial p_4} = \frac{3v_1^2 - 6\pi_1\pi_2 - \pi_2^2}{2p_1p_2\pi_1\pi_2} + \frac{(\pi_2 - \pi_1)\left(2p_2\pi_1^2 - p_1\pi_1\pi_2 + p_2\pi_2^2 + 2p_1\pi_1^2\right)}{2p_1p_2\pi_1\pi_2(\pi_2p_1 - \pi_1p_2)}$$

$$\frac{\partial \lambda_1}{\partial p_4} = \frac{4p_1\pi_1^2\pi_2 - 6p_1\pi_1\pi_2^2 + 8p_2\pi_1^2\pi_2 - p_1\pi_2^3 - 5p_2\pi_1^2\pi_2 + p_2\pi_2^3 - 2p_1\pi_1^3}{2p_1p_2\pi_1\pi_2(\pi_2p_1 - \pi_1p_2)}, \tag{32}$$

where $\pi_2p_1 \neq \pi_1p_2$ and $p_1p_2\pi_1\pi_2 > 0$.

Let, $\pi_1 = \pi_2 = \pi$ in (32), and then we get,

$$\frac{\partial \lambda_1}{\partial p_4} = -\frac{5p_1 - 4p_2}{2p_1p_2(p_1 - p_2)}, \tag{33}$$

where $p_1 \neq p_2$.

If $p_1 > p_2$ we have from (33),

$$\frac{\partial \lambda_1}{\partial p_4} < 0. \tag{34}$$

The inequality (34) specifies that if the price p_4 of the product m_4 upsurges, the level of marginal utility will decline. Therefore, if the price of per unit of product m_4 surges \$1.00, the level of purchase will decline exactly λ_1 units. Therefore, product m_4 has some substitutes, such as commodities m_1, m_2, m_3; and buyers move to substitutes when price of product m_4 raises [Islam et al., 2010, Mohajan, 2021a].

If $p_1 < p_2$, but $5p_1 > 4p_2$; then we have from (33),

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$$\frac{\partial \lambda_1}{\partial p_4} > 0 . \tag{35}$$

The inequality (35) shows that if the price p_4 of goods m_4 upsurges, the marginal utility also upsurges. Hence, if the price of per unit of product m_4 increases \$1.00, the level of buying will growth exactly λ_1 units. Therefore, product m_4 is a good of best quality, and it has no substitutes [Mohajan & Mohajan, 2023b]. In this case, commodities m_1, m_2, and m_3 seem inferior goods. The organization should try to produce more of good m_4, and should decrease the production rate of commodities m_1, m_2, and m_3 for the sustainability of the organization.

From (33) we have realized that,

$$\frac{\partial \lambda_1}{\partial p_4} \neq 0 . \tag{36}$$

Therefore, $5p_1 \neq 4p_2$, and also $p_1 \neq p_2$, i.e., commodities m_1 and m_2 are completely different.

We discuss the properties of λ_2 when unit price p_3 of commodity m_3 rises. Considering T_{23} from (17) we find [Islam et al., 2011; Mohajan et al., 2013],

$$\frac{\partial \lambda_2}{\partial p_3} = \frac{m_3}{|J|}[C_{12}] + \frac{\lambda_1}{|J|}[C_{52}]$$

$$= \frac{m_3}{|J|}\text{Cofactor of } C_{12} + \frac{\lambda_1}{|J|}\text{Cofactor of } C_{52}$$

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$$= -\frac{m_3}{|J|}\begin{vmatrix} 0 & -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ -B_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -B_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -B_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -B_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix}$$

$$-\frac{\lambda_1}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4 \\ -B_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -B_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -B_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix}$$

$$= -\frac{m_3}{|J|}\left\{ \Pi_1 \begin{vmatrix} -B_1 & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ -B_2 & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ -B_3 & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} \\ -B_4 & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{vmatrix} - \Pi_2 \begin{vmatrix} -B_1 & \Lambda_{11} & \Lambda_{13} & \Lambda_{14} \\ -B_2 & \Lambda_{21} & \Lambda_{23} & \Lambda_{24} \\ -B_3 & \Lambda_{31} & \Lambda_{33} & \Lambda_{34} \\ -B_4 & \Lambda_{41} & \Lambda_{43} & \Lambda_{44} \end{vmatrix} \right.$$

$$\left. + \Pi_3 \begin{vmatrix} -B_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{14} \\ -B_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{24} \\ -B_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{34} \\ -B_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{44} \end{vmatrix} - \Pi_4 \begin{vmatrix} -B_1 & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ -B_2 & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ -B_3 & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \\ -B_4 & \Lambda_{41} & \Lambda_{42} & \Lambda_{43} \end{vmatrix} \right\}$$

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$$-\frac{\lambda_1}{|J|}\left\{-B_1\begin{vmatrix}-B_1 & -B_2 & -B_3 & -B_4\\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + B_2\begin{vmatrix}-B_1 & -B_2 & -B_3 & -B_4\\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix}\right.$$

$$\left. - B_4\begin{vmatrix}-B_1 & -B_2 & -B_3 & -B_4\\ -\Pi_1 & -\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14}\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\end{vmatrix}\right\}$$

$$= -\frac{m_3}{|J|}\left[\Pi_1\left\{-B_1\begin{vmatrix}\Lambda_{22} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{32} & \Lambda_{33} & \Lambda_{34}\\ \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} - \Lambda_{12}\begin{vmatrix}-B_2 & \Lambda_{23} & \Lambda_{24}\\ -B_3 & \Lambda_{33} & \Lambda_{34}\\ -B_4 & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + \Lambda_{13}\begin{vmatrix}-B_2 & \Lambda_{22} & \Lambda_{24}\\ -B_3 & \Lambda_{32} & \Lambda_{34}\\ -B_4 & \Lambda_{42} & \Lambda_{44}\end{vmatrix}\right.\right.$$

$$\left.\left. - \Lambda_{14}\begin{vmatrix}-B_2 & \Lambda_{22} & \Lambda_{23}\\ -B_3 & \Lambda_{32} & \Lambda_{33}\\ -B_4 & \Lambda_{42} & \Lambda_{43}\end{vmatrix}\right\} - \Pi_2\left\{-B_1\begin{vmatrix}\Lambda_{21} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{33} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + \Lambda_{13}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{24}\\ -B_3 & \Lambda_{31} & \Lambda_{34}\\ -B_4 & \Lambda_{41} & \Lambda_{44}\end{vmatrix}\right.$$

$$\left. - \Lambda_{14}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{23}\\ -B_3 & \Lambda_{31} & \Lambda_{33}\\ -B_4 & \Lambda_{41} & \Lambda_{43}\end{vmatrix}\right\} + \Pi_3\left\{-B_1\begin{vmatrix}\Lambda_{21} & \Lambda_{22} & \Lambda_{24}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{34}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{44}\end{vmatrix} + \Lambda_{12}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{24}\\ -B_3 & \Lambda_{31} & \Lambda_{34}\\ -B_4 & \Lambda_{41} & \Lambda_{44}\end{vmatrix}\right.$$

$$\left. - \Lambda_{14}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{22}\\ -B_3 & \Lambda_{31} & \Lambda_{32}\\ -B_4 & \Lambda_{41} & \Lambda_{42}\end{vmatrix}\right\} - \Pi_4\left\{-B_1\begin{vmatrix}\Lambda_{21} & \Lambda_{22} & \Lambda_{23}\\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43}\end{vmatrix} + \Lambda_{12}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{23}\\ -B_3 & \Lambda_{31} & \Lambda_{33}\\ -B_4 & \Lambda_{41} & \Lambda_{43}\end{vmatrix}\right.$$

$$\left.\left. - \Lambda_{13}\begin{vmatrix}-B_2 & \Lambda_{21} & \Lambda_{22}\\ -B_3 & \Lambda_{31} & \Lambda_{32}\\ -B_4 & \Lambda_{41} & \Lambda_{42}\end{vmatrix}\right\}\right] - \frac{\lambda_1}{|J|}\left[-B_1\left\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} + B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{21} & \Lambda_{23} & \Lambda_{24}\\ \Lambda_{41} & \Lambda_{43} & \Lambda_{44}\end{vmatrix}\right.\right.$$

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$$-B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{24}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{44}\end{vmatrix} +B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43}\end{vmatrix}\Bigg\} +B_2\Bigg\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{12} & \Lambda_{13} & \Lambda_{14}\\ \Lambda_{42} & \Lambda_{43} & \Lambda_{44}\end{vmatrix}$$

$$+B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{11} & \Lambda_{13} & \Lambda_{14}\\ \Lambda_{41} & \Lambda_{43} & \Lambda_{44}\end{vmatrix} -B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{11} & \Lambda_{12} & \Lambda_{14}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{44}\end{vmatrix} +B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{11} & \Lambda_{12} & \Lambda_{13}\\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43}\end{vmatrix}\Bigg\}$$

$$-B_4\Bigg\{-B_1\begin{vmatrix}-\Pi_2 & -\Pi_3 & -\Pi_4\\ \Lambda_{12} & \Lambda_{13} & \Lambda_{14}\\ \Lambda_{22} & \Lambda_{23} & \Lambda_{24}\end{vmatrix} +B_2\begin{vmatrix}-\Pi_1 & -\Pi_3 & -\Pi_4\\ \Lambda_{11} & \Lambda_{13} & \Lambda_{14}\\ \Lambda_{21} & \Lambda_{23} & \Lambda_{24}\end{vmatrix} -B_3\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_4\\ \Lambda_{11} & \Lambda_{12} & \Lambda_{14}\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{24}\end{vmatrix}$$

$$+B_4\begin{vmatrix}-\Pi_1 & -\Pi_2 & -\Pi_3\\ \Lambda_{11} & \Lambda_{12} & \Lambda_{13}\\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23}\end{vmatrix}\Bigg\}\Bigg]$$

$$=-\frac{m_3}{|J|}\{-B_1\Pi_1\Lambda_{23}\Lambda_{24}\Lambda_{34} -B_1\Pi_1\Lambda_{23}\Lambda_{24}\Lambda_{34} -B_2\Pi_1\Lambda_{12}\Lambda_{34}^2 +B_4\Pi_1\Lambda_{12}\Lambda_{23}\Lambda_{34}$$

$$+B_3\Pi_1\Lambda_{12}\Lambda_{24}\Lambda_{34} +B_2\Pi_1\Lambda_{13}\Lambda_{24}\Lambda_{34} -B_3\Pi_1\Lambda_{13}\Lambda_{24}^2 +B_4\Pi_1\Lambda_{13}\Lambda_{23}\Lambda_{24}$$

$$-B_2\Pi_1\Lambda_{14}\Lambda_{23}\Lambda_{34} +B_3\Pi_1\Lambda_{14}\Lambda_{23}\Lambda_{24} -B_4\Pi_1\Lambda_{14}\Lambda_{23}^2 -B_1\Pi_2\Lambda_{12}\Lambda_{34}^2$$

$$-B_1\Pi_2\Lambda_{14}\Lambda_{23}\Lambda_{34} -B_1\Pi_2\Lambda_{13}\Lambda_{24}\Lambda_{34} -B_2\Pi_2\Lambda_{13}\Lambda_{14}\Lambda_{34} +B_4\Pi_2\Lambda_{12}\Lambda_{13}\Lambda_{34}$$

$$-B_4\Pi_2\Lambda_{13}^2\Lambda_{24} +B_4\Pi_2\Lambda_{13}\Lambda_{14}\Lambda_{24} -B_2\Pi_2\Lambda_{13}\Lambda_{14}\Lambda_{34} +B_3\Pi_2\Lambda_{12}\Lambda_{14}\Lambda_{34}$$

$$-B_3\Pi_2\Lambda_{14}^2\Lambda_{23} +B_4\Pi_2\Lambda_{13}\Lambda_{14}\Lambda_{23} +B_1\Pi_3\Lambda_{12}\Lambda_{24}\Lambda_{34} -B_1\Pi_3\Lambda_{13}\Lambda_{24}^2$$

$$+B_1\Pi_3\Lambda_{14}\Lambda_{23}\Lambda_{24} +B_2\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{34} -B_4\Pi_3\Lambda_{12}^2\Lambda_{34} -B_3\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{24}$$

$$+B_4\Pi_3\Lambda_{12}\Lambda_{13}\Lambda_{24} +B_2\Pi_3\Lambda_{13}\Lambda_{14}\Lambda_{24} -B_2\Pi_3\Lambda_{14}^2\Lambda_{23} -B_3\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{24}$$

$$+B_4\Pi_3\Lambda_{12}\Lambda_{14}\Lambda_{23} +B_1\Pi_4\Lambda_{12}\Lambda_{23}\Lambda_{34} +B_1\Pi_4\Lambda_{13}\Lambda_{23}\Lambda_{24} -B_1\Pi_4\Lambda_{14}\Lambda_{23}^2$$

$$+B_2\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{34} -B_3\Pi_4\Lambda_{12}^2\Lambda_{34} +B_3\Pi_4\Lambda_{12}\Lambda_{14}\Lambda_{23} -B_4\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{23}$$

$$-B_2\Pi_4\Lambda_{13}^2\Lambda_{24} +B_2\Pi_4\Lambda_{13}\Lambda_{14}\Lambda_{23} +B_3\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{24} -B_4\Pi_4\Lambda_{12}\Lambda_{13}\Lambda_{23}\}$$

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$$-\frac{\lambda_1}{|J|}\{B_1^2\Pi_2\Lambda_{24}\Lambda_{34} - B_1^2\Pi_3\Lambda_{24}^2 + B_1^2\Pi_4\Lambda_{23}\Lambda_{24} - B_1B_2\Pi_1\Lambda_{24}\Lambda_{34} + B_1B_2\Pi_3\Lambda_{14}\Lambda_{24}$$
$$+ B_1B_2\Pi_4\Lambda_{12}\Lambda_{34} - B_1B_2\Pi_4\Lambda_{14}\Lambda_{23} + B_1B_3\Pi_1\Lambda_{24}^2 - B_1B_3\Pi_2\Lambda_{14}\Lambda_{24}$$
$$+ B_1B_3\Pi_4\Lambda_{12}\Lambda_{24} - B_1B_4\Pi_1\Lambda_{23}\Lambda_{24} - B_1B_4\Pi_2\Lambda_{12}\Lambda_{34} + B_1B_4\Pi_2\Lambda_{14}\Lambda_{23}$$
$$+ B_1B_4\Pi_3\Lambda_{12}\Lambda_{24} - B_1B_2\Pi_2\Lambda_{14}\Lambda_{34} + B_1B_3\Pi_3\Lambda_{14}\Lambda_{24} + B_1B_2\Pi_4\Lambda_{12}\Lambda_{34}$$
$$- B_1B_2\Pi_4\Lambda_{13}\Lambda_{24} + B_2^2\Pi_1\Lambda_{14}\Lambda_{34} - B_2^2\Pi_3\Lambda_{14}^2 + B_2^2\Pi_4\Lambda_{13}\Lambda_{14} - B_2B_3\Pi_1\Lambda_{14}\Lambda_{24}$$
$$+ B_2B_3\Pi_2\Lambda_{14}^2 - B_2B_3\Pi_4\Lambda_{12}\Lambda_{14} - B_2B_4\Pi_1\Lambda_{12}\Lambda_{34} + B_2B_4\Pi_1\Lambda_{13}\Lambda_{24}$$
$$- B_2B_4\Pi_2\Lambda_{13}\Lambda_{14} + B_2B_4\Pi_3\Lambda_{12}\Lambda_{14} - B_1B_4\Pi_2\Lambda_{13}\Lambda_{24} + B_1B_4\Pi_2\Lambda_{14}\Lambda_{23}$$
$$+ B_1B_4\Pi_3\Lambda_{12}\Lambda_{24} - B_1B_4\Pi_4\Lambda_{12}\Lambda_{23} + B_2B_4\Pi_1\Lambda_{13}\Lambda_{24} - B_2B_4\Pi_1\Lambda_{14}\Lambda_{23}$$
$$+ B_2B_4\Pi_3\Lambda_{12}\Lambda_{14} - B_2B_4\Pi_4\Lambda_{12}\Lambda_{13} - B_3B_4\Pi_1\Lambda_{12}\Lambda_{24} - B_3B_4\Pi_2\Lambda_{12}\Lambda_{14}$$
$$+ B_3B_4\Pi_4\Lambda_{12}^2 - B_4^2\Pi_1\Lambda_{12}\Lambda_{23} - B_4^2\Pi_2\Lambda_{12}\Lambda_{13} + B_4^2\Pi_2\Lambda_{12}^2\}$$
$$\frac{\partial\lambda_2}{\partial p_3} = -\frac{m_3}{|J|}\{-2p_1\pi_1m_1^3m_2m_3m_4 - p_2\pi_1m_1^2m_2^2m_3m_4 + p_4\pi_1m_1^2m_2m_3m_4^2$$
$$+ p_3\pi_1m_1^2m_2m_3^2m_4 + p_2\pi_1m_1^2m_2^2m_3m_4 - p_3\pi_1m_1^2m_2m_3^2m_4 + p_4\pi_1m_1^2m_2m_3m_4^2$$
$$- p_2\pi_1m_1^2m_2^2m_3m_4 - p_3\pi_1m_1^2m_2m_3^2m_4 - p_4\pi_1m_1^2m_2m_3m_4^2 - p_1\pi_2m_1^2m_2^2m_3m_4$$
$$- p_1\pi_2m_1^2m_2^2m_3m_4 - p_1\pi_2m_1^2m_2^2m_3m_4 - p_2\pi_2m_1m_2^3m_3m_4 + p_4\pi_2m_1m_2^2m_3m_4^2$$
$$- p_4\pi_2m_1m_2^2m_3m_4^2 + p_4\pi_2m_1m_2^2m_3^2m_4 - p_2\pi_2m_1m_2^3m_3m_4 + p_3\pi_2m_1m_2^2m_3^2m_4$$
$$- p_3\pi_2m_1m_2^2m_3^2m_4 + p_4\pi_2m_1m_2^2m_3m_4^2 + p_1\pi_3m_1^2m_2m_3^2m_4 - p_1\pi_3m_1^2m_2m_3^2m_4$$
$$+ p_1\pi_3m_1^2m_2m_3^2m_4 - p_2\pi_3m_1m_2^2m_3^2m_4 - p_4\pi_3m_1m_2m_3^2m_4^2 - p_3\pi_3m_1m_2m_3^3m_4$$
$$+ p_4\pi_3m_1m_2m_3^2m_4^2 + p_2\pi_3m_1m_2^2m_3^2m_4 - p_2\pi_3m_1m_2^2m_3^2m_4 - p_3\pi_3m_1m_2m_3^3m_4$$
$$+ p_4\pi_3m_1m_2m_3^2m_4^2 + p_1\pi_4m_1^2m_2m_3m_4^2 + p_1\pi_4m_1^2m_2m_3m_4^2 - p_1\pi_4m_1^2m_2m_3m_4^2$$
$$+ p_2\pi_4m_1m_2^2m_3m_4^2 - p_3\pi_4m_1m_2m_3^2m_4^2 + p_3\pi_4m_1m_2m_3^2m_4^2 - p_4\pi_4m_1m_2m_3m_4^3$$
$$- p_4\pi_2m_1m_2^2m_3m_4^2 - p_2\pi_4m_1m_2^2m_3m_4^2 + p_2\pi_4m_1m_2^2m_3m_4^2 + p_3\pi_4m_1m_2m_3^2m_4^2$$
$$- p_4\pi_4m_1m_2m_3m_4^3\} - \frac{\lambda_1}{|J|}\{p_1^2\pi_2m_1^2m_2m_3 - p_1^2\pi_3m_1^2m_3^2 + p_1^2\pi_4m_1^2m_3m_4$$

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$$- p_1 p_2 \pi_1 m_1^2 m_2 m_3 \quad + p_1 p_2 \pi_3 m_1 m_2 m_3^2 \quad + p_1 p_2 \pi_4 m_1 m_2 m_3 m_4 \quad - p_1 p_2 \pi_4 m_1 m_2 m_3 m_4$$
$$+ p_1 p_3 \pi_1 m_1^2 m_3^2 \quad - p_1 p_3 \pi_2 m_1 m_2 m_3^2 \quad + p_1 p_3 \pi_4 m_1 m_3^2 m_4 \quad - p_1 p_4 \pi_1 m_1^2 m_3 m_4$$
$$- p_1 p_4 \pi_2 m_1 m_2 m_3 m_4 \quad + p_1 p_4 \pi_2 m_1 m_2 m_3 m_4 \quad + p_1 p_4 \pi_3 m_1 m_3^2 m_4 \quad - p_1 p_2 \pi_2 m_1 m_2^2 m_3$$
$$+ p_1 p_3 \pi_3 m_1 m_2 m_3^2 \quad + p_1 p_2 \pi_4 m_1 m_2 m_3 m_4 \quad - p_1 p_2 \pi_4 m_1 m_2 m_3 m_4 \quad + p_2^2 \pi_1 m_1 m_2^2 m_3$$
$$- p_2^2 \pi_3 m_2^2 m_3^2 \quad + p_2^2 \pi_4 m_2^2 m_3 m_4 \quad + p_2 p_3 \pi_2 m_2^2 m_3^2 \quad - p_2 p_3 \pi_4 m_2 m_3^2 m_4$$
$$- p_2 p_4 \pi_1 m_1 m_2 m_3 m_4 \quad + p_2 p_4 \pi_1 m_1 m_2 m_3 m_4 \quad - p_2 p_4 \pi_2 m_2^2 m_3 m_4 \quad + p_2 p_4 \pi_3 m_2 m_3^2 m_4$$
$$- p_1 p_4 \pi_2 m_1 m_2 m_3 m_4 \quad + p_1 p_4 \pi_2 m_1 m_2 m_3 m_4 \quad + p_1 p_4 \pi_3 m_1 m_3^2 m_4 \quad - p_1 p_4 \pi_4 m_1 m_3 m_4^2$$
$$+ p_2 p_4 \pi_1 m_1 m_2 m_3 m_4 \quad - p_2 p_4 \pi_1 m_1 m_2 m_3 m_4 \quad + p_2 p_4 \pi_3 m_2 m_3^2 m_4 \quad - p_2 p_4 \pi_4 m_2 m_3 m_4^2$$
$$- p_3 p_4 \pi_1 m_1 m_3^2 m_4 \quad - p_3 p_4 \pi_2 m_2 m_3^2 m_4 \quad + p_3 p_4 \pi_4 m_2 m_3^2 m_4^2 \quad - p_4^2 \pi_1 m_1 m_3 m_4^2$$
$$- p_4^2 \pi_2 m_2 m_3 m_4^2$$
$$+ p_4^2 \pi_2 m_3^2 m_4^2 \Big\}. \tag{37}$$

Using $\pi_3 = \pi_1$ and $\pi_4 = \pi_2$; and $p_3 = p_1$ and $p_4 = p_2$; and also $m_3 = m_1$ and $m_4 = m_2$ in (37) we get,

$$\frac{\partial \lambda_2}{\partial p_3} = -\frac{m_1}{|J|}\left\{-4 p_1 \pi_1 m_1^4 m_2^2 \quad - p_1 \pi_2 m_1^3 m_2^3 \quad + p_2 \pi_2 m_1^3 m_2^3 \quad - 3 p_2 \pi_2 m_1^2 m_2^4\right\}$$
$$-\frac{\lambda_1}{|J|}\left\{2 p_1^2 \pi_2 m_1^3 m_2 \; + p_1 p_2 \pi_1 m_1^3 m_2 \; + p_1^2 \pi_1 m_1^3 m_2 \; - 2 p_1 p_2 \pi_2 m_1^2 m_2^2 \; - p_1 p_2 \pi_1 m_1 m_2^3\right.$$
$$- p_2^2 \pi_2 m_1 m_2^3$$
$$\left. + p_2^2 \pi_2 m_1^2 m_2^2 \right\}. \tag{38}$$

From (20) using value of λ_1 , and also using $|J| = |H| = -2 p_1 p_2 \pi_1 \pi_2$ from (8), and considering $m_1 = m_2 = m_3 = m_4 = 1$ in (38) we get,

$$\frac{\partial \lambda_2}{\partial p_3} = \frac{5 p_1 \pi_1 + 2 p_2 \pi_2}{2 p_1 p_2 \pi_1 \pi_2} + \frac{(\pi_2 - \pi_1)\left(2 p_1^2 \pi_2 - 2 p_1 p_2 \pi_2 + p_1^2 \pi_1\right)}{2 p_1 p_2 \pi_1 \pi_2 (\pi_2 p_1 - \pi_1 p_2)}$$
$$\frac{\partial \lambda_2}{\partial p_3} = \frac{4 p_1^2 n_1 \pi_2 + 2 p_1 p_2 \pi_1 \pi_2 + 2 p_1^2 \pi_2^2 - p_1^2 \pi_1^2 - 5 p_1 p_2 \pi_1^2 - 2 p_2^2 \pi_1 \pi_2}{2 p_1 p_2 \pi_1 \pi_2 (\pi_2 p_1 - \pi_1 p_2)} \tag{39}$$

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where $\pi_2 p_1 \neq \pi_1 p_2$ and $2p_1 p_2 \pi_1 \pi_2 > 0$.

Let us consider $\pi_1 = \pi_2 = \pi$ in (39) then we obtain,

$$\frac{\partial \lambda_2}{\partial p_3} = \frac{5p_1 + 2p_2}{2p_1 p_2 \pi} > 0. \tag{40}$$

The inequality (40) informs that if the price p_3 of the goods m_3 upsurges, marginal utility also raises. Therefore, if the price of per unit of product m_3 surges \$1.00, the level of buying will upsurge exactly λ_2 units and the organization should surge the production of product m_3 to achieve maximum profit [Mohajan, 2022].

6. Conclusions

In this paper we studies economic predictions among Lagrange multipliers and unit commodity prices for utility maximization. In the article we used a pair of constraints. We have understood that the Lagrange multipliers method is a very useful and powerful both for the consumers and producers. In the article we have worked with 16 variables and consequently, we were flexible in some cases. In the sensitivity analysis we have tried with various commodity prices and have tried our best to provide accurate prediction. We believe that future researchers can contribute more efforts in this field to enrich the mathematical economic research area.

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